

## 複利和年金

### 問答題

- 單利的計算基礎僅為每期的原始本金金額；複利計算的基礎則為期初的本金金額加上先前各期累積的利息，若期間在兩期以上，則本金所產生的利息會加入本金繼續再衍生新的利息，亦即利上加利。  
因此，在利率條件相同的情況下，複利計算的結果，金額會較單利計算結果為大。
- 終值為某筆或多筆投資金額，經由複利計算後，在未來特定日所累積變成的金額。現值則是未來某筆或多筆金額，經由複利計算後，在今日折現後的金額。
- 相等間隔時間連續支付（或收取）相等金額，且每期計息之利率也相同，即所謂的年金（annuity）。由於各期金額的收付可於期初或期末為之，因此年金又區分為二類，於期末收付者，稱為普通年金（ordinary annuity）；於期初收付者，稱為到期年金（annuity due）。
- 所謂遞延年金（deferred annuities），係指於若干期後才發生收付的年金。例如遞延 3 年之五年普通年金，意謂前三年並無金額收付的發生，而第一期的收付是發生於第四年底，且連續 5 年。
- 債券的面額、票面利率、債券的發行的日期、付息日期、到期日與發行時之市場利率。

### 選擇題

- |         |         |         |
|---------|---------|---------|
| 1. (C)  | 2. (D)  | 3. (D)  |
| 4. (D)  | 5. (B)  | 6. (B)  |
| 7. (C)  | 8. (B)  | 9. (C)  |
| 10. (B) | 11. (A) | 12. (C) |
| 13. (B) | 14. (C) | 15. (A) |
| 16. (D) | 17. (C) | 18. (C) |
| 19. (C) |         |         |

## 練習題

1. (1)  $\$100,000 + \$100,000 \times 4\% \times 6 = \$124,000$   
 (2)  $\$100,000 \times (1 + 0.04)^6 = \$100,000 \times 1.26532 = \$126,532$
  
2. (1)  $\$60,000 \times (\text{Future Value of } 1, 4 \text{ periods, } 12\%)$   
 $= \$60,000 \times 1.57352 = \$94,411$   
  
 (2)  $\$60,000 \times (\text{Future Value of } 1, 8 \text{ periods, } 6\%)$   
 $= \$60,000 \times 1.59385 = \$95,631$   
  
 (3)  $\$60,000 \times (\text{Future Value of } 1, 16 \text{ periods, } 3\%)$   
 $= \$60,000 \times 1.60471 = \$96,283$
  
3.  $\$1,300,000 \times (\text{Present Value of } 1, 7 \text{ periods, } 8\%)$   
 $= \$1,300,000 \times 0.58349 = \$758,537$
  
4. (1)  $\$130,000 \times (\text{Future Value of } 1, 6 \text{ periods, } 12\%)$   
 $= \$130,000 \times 1.97382 = \$256,597$   
  
 (2)  $\$45,000 \times (\text{Present Value of } 1, 2 \text{ periods, } 10\%)$   
 $= \$45,000 \times 0.82645 = \$37,190$   
  
 (3)  $\$30,000 \times (\text{Future Value of } 1, 6 \text{ periods, } 2\%)$   
 $= \$30,000 \times 1.12616 = \$33,785$   
  
 (4)  $\$250,000 \times (\text{Present Value of } 1, 5 \text{ periods, } 15\%)$   
 $= \$250,000 \times 0.49718 = \$124,295$
  
5.  $\$10,000 \times (\text{Future Value of an Ordinary Annuity, } 6 \text{ periods, } 5\%)$   
 $= \$10,000 \times 6.80191 = \$68,020$
  
6.  $\$14,000,000 \div (\text{Future Value of an Ordinary Annuity, } 11 \text{ periods, } 8\%)$   
 $= \$14,000,000 \div 16.64549 = \$841,069$
  
7.  $\$1,000,000 \div (\text{Future Value of an Annuity Due, } 6 \text{ periods, } 4\%)$   
 $= \$1,000,000 \div 6.89829 = \$144,963$
  
8.  $\$108,871 \times (\text{Future Value of an Annuity Due, } N \text{ periods, } 10\%) = \$840,000$   
 $(\text{Future Value of an Annuity Due, } N \text{ periods, } 10\%) = 7.71555$   
 $(\text{Future Value of an Ordinary Annuity, } N \text{ periods, } 10\%) \times 1.1 = 7.71555$   
 $\text{故}(\text{Future Value of an Ordinary Annuity, } N \text{ periods, } 10\%) = 7.01414$   
 $\text{When } N=5 \rightarrow 6.105100 \text{ (不足); When } N=6 \rightarrow 7.715610$   
 $\text{故花花要存 6 年方可購買價值\$840,000 的東西，即 } N = 6 \text{ (年)}$

9.  $\$500,000 \times 4\% \times (\text{Present Value of an Ordinary Annuity, 8 periods, } 3\%) + \$500,000 \times (\text{Present Value of 1, 8 periods, } 3\%)$   
 $= \$20,000 \times 7.01969 + \$500,000 \times 0.78941 = \$535,099$

10.  $\$125,000 \times (\text{Present Value of an Ordinary Annuity, 5 periods, } 12\%)$   
 $\$125,000 \times 3.60478 = \$450,597$

11. 店面甲： $\$50,000,000$

店面乙：

租金現值： $\$7,000,000 \times (\text{Present Value of an Annuity Due, 20 periods, } 15\%)$   
 $= \$7,000,000 \times 7.19823 = \underline{\$50,387,610}$

店面丙：

租金現值 =  $\$860,000 \times (\text{Present Value of an Ordinary Annuity, 20 periods, } 15\%)$   
 $= \$860,000 \times 6.25933 = \$5,383,024$

店面丙之淨現值 =  $\$55,000,000 - \$5,383,024 = \underline{\$49,616,976}$

由於店面丙之現值最低，故哲普公司應選擇店面丙。

12.  $\$56,000 \times (\text{Present Value of an Annuity Due, 36 periods, } 2\%)$   
 $= \$56,000 \times 25.99862 = \$1,455,923$

13.  $\$7,000 \times (\text{Present Value of an Annuity Due, 6 periods, } 2.5\%)$   
 $= \$7,000 \times 5.64583 = \$39,521$

分期付款現值 \$39,521 大於現購價 \$39,000，故直接購買較划算。

14.  $\$340,000 \times (\text{Present Value of an Annuity Due, 7 periods, } 8\%)$   
 $= \$340,000 \times 5.62288 = \$1,911,779$   
 $\$1,911,779 \times (\text{Present Value of an 1, 5 periods, } 8\%)$   
 $= \$1,911,779 \times 0.68058 = \$1,301,119$   
 $\$1,301,119 \div (\text{Future Value of an Ordinary Annuity, 8 periods, } 8\%)$   
 $= \$1,301,119 \div 10.63663 = \$122,324$

15.  $\$70,000 \times (\text{Present Value of an Annuity Due, 10 periods, } 4\%) \times (\text{Present Value of 1, 2 periods, } 8\%)$   
 $= \$70,000 \times 8.43533 \times 0.85734 = \$506,236$

現購價 =  $\$150,000 + \$506,236 = \$656,236$

**應用問題**

1.

年度	現金流量	折現因子	現值
×2	\$579,481	0.89286	\$ 517,395
×3	588,232	0.79719	468,933
×4	517,526	0.71178	368,365
×5	522,483	0.63552	332,048
×6	498,724	0.56743	282,991
×7	483,557	0.50663	244,984
資產之使用價值			<u>\$ 2,214,716</u>

2. 方式一

$$\$450,000 \div \$100,000 = 4.5$$

查表 Future Value of an Ordinary Annuity, 4%

$$4 \text{ periods} = 4.24646 ; 5 \text{ periods} = 5.41632$$

第五次付款達到目標—×3 年 6 月 30 日

**方式二**

$$\$450,000 \div \$49,500 = 9.09$$

查表 Future Value of an Ordinary Annuity, 2%

$$8 \text{ periods} = 8.58297 ; 9 \text{ periods} = 9.75463$$

第九次付款達到目標—×3 年 3 月 31 日

應選擇方式二，可於×3 年 3 月 31 日達到目標。

3.  $\$500,000 \times (\text{Future Value of } 1, 20 \text{ periods}, 4\%)$ 

$$= \$500,000 \times 2.19112 = \$1,095,560$$

$$\$10,000,000 - \$1,095,560 = \$8,904,440$$

$$\$8,904,440 \div (\text{Future Value of an Ordinary Annuity, 10 periods, } 4\%)$$

$$= \$8,904,440 \div 12.00611 = \$741,659$$

4. 方法一

$$\$1,400,000 \times (\text{Future Value of } 1, 5 \text{ periods}, 15\%)$$

$$= \$1,400,000 \times 2.01136 = \underline{\$2,815,904}$$

**方法二**

$$\$100,000 \times (\text{Future Value of an Annuity Due, 20 periods, } 3\%)$$

$$= \$100,000 \times 27.67649 = \underline{\$2,767,649}$$

**方法三**

$$\begin{aligned} & \$400,000 \times (\text{Future Value of an Annuity Due, 5 periods, } 12\%) \\ & = \$400,000 \times 7.11519 = \underline{\$2,846,076} \end{aligned}$$

由於方法三之終值金額最高，書豪應選擇方法三。

5.  $\$2,000,000 \times 2\% \times (\text{Present Value of an Ordinary Annuity, 12 periods, } 3\%) + \$2,000,000 \times (\text{Present Value of 1, 12 periods, } 3\%)$   
 $= \$40,000 \times 9.95400 + \$2,000,000 \times 0.70138 = \underline{\$1,800,920}$

**6. 方法一**

$$\begin{aligned} & \$6,000 + \$16,000 \times (\text{Present Value of an 1, 2 periods, } 5\%) \\ & = \$6,000 + \$16,000 \times 0.90703 = \underline{\$20,512} \end{aligned}$$

**方法二**

$$\begin{aligned} & \$11,000 \times (\text{Present Value of an Ordinary Annuity, 2 periods, } 5\%) \\ & = \$11,000 \times 1.85941 = \underline{\$20,454} \end{aligned}$$

由於方法二之現值較低，八戒應選擇方法二。

**7. 方案 A**

$$\underline{\$300,000}$$

**方案 B**

$$\begin{aligned} & \$40,000 \times (\text{Present Value of an Annuity Due, 10 periods, } 8\%) \\ & = \$40,000 \times 7.24689 = \underline{\$289,876} \end{aligned}$$

由於方案 B 之現值較低，蘋果公司應選擇方案 B。

**8. A 公司：**

$$\begin{aligned} & \$50,000 \times (\text{Present Value of an Annuity Due, 10 periods, } 10\%) \\ & = \$50,000 \times 6.75902 = \underline{\$337,951} \end{aligned}$$

**B 公司：**

$$\begin{aligned} & \$100,000 + \$25,000 \times (\text{Present Value of an Annuity Due, 16 periods, } 5\%) - \$100,000 \times (\text{Present Value of 1, 10 periods, } 10\%) \\ & = \$100,000 + \$25,000 \times 11.37966 - \$100,000 \times 0.38554 = \underline{\$345,938} \end{aligned}$$

**C 公司：**

$$\begin{aligned} & \$250,000 + \$9,000 \times (\text{Present Value of an Annuity Due, 10 periods, } 10\%) \\ & = \$250,000 + \$9,000 \times 6.75902 = \underline{\$310,831} \end{aligned}$$

C 清潔公司收款之現值最低，故諾貝爾公司應選擇與 C 清潔公司簽約。

**9. A 買家：\\$23,000,000**

$$\begin{aligned} \text{B 買家} : & \$15,000,000 + \$9,500,000 \times (\text{Present Value of an 1, 2 periods, } 12\%) \\ & = \$15,000,000 + \$9,500,000 \times 0.79719 = \underline{\$22,573,305} \end{aligned}$$

$$\begin{aligned} \text{C 買家} : & \$5,400,000 \times (\text{Present Value of an Annuity Due, 15 periods, } 12\%) \times (\text{Present Value of } \\ & 1, 5 \text{ periods, } 12\%) = \$5,400,000 \times 7.62817 \times 0.56743 = \underline{\$23,373,644} \end{aligned}$$

C 買家所支付款項之現值最高，故應將資產銷售給 C 買家。

#### 10. $\times 5$ 年底之金額

$$\begin{aligned} & = \$5,050,425 \times (\text{Future Value of an 1, 4 periods, } 8\%) \\ & = \$5,050,425 \times 1.36049 = \$6,871,053 \end{aligned}$$

$\times 6$  年~ $\times 10$  年領取金額於  $\times 6$  年初之現值

$$\begin{aligned} & = \$600,000 \times (\text{Present Value of an Annuity Due, 5 periods, } 8\%) \\ & = \$600,000 \times 4.31213 = \$2,587,278 \end{aligned}$$

$\times 11$  年~ $\times 25$  年領取金額於  $\times 6$  年初之現值

$$\$6,871,053 - \$2,587,278 = \$4,283,775$$

$\times 11$  年~ $\times 25$  年領取金額

$$\begin{aligned} & = \$4,283,775 \div [(\text{到期年金現值 Present Value of an Annuity Due, 30 periods, } 4\%) \times (\text{Present} \\ & \text{Value of an 1, 5 periods, } 8\%)] \\ & = \$4,283,775 \div (17.98371 \times 0.68058) = \underline{\$350,000} \end{aligned}$$

#### 11. 本利和之金額

$$\begin{aligned} & = \$200,000 \times (\text{Future Value of an 1, 6 periods, } 1\%) \times (\text{Future Value of an 1, 8 periods, } 1.5\%) \times \\ & (\text{Future Value of an 1, 6 periods, } 2.5\%) + \$250,000 \times (\text{Future Value of an 1, 8 periods, } 1.5\%) \times \\ & (\text{Future Value of an 1, 6 periods, } 2.5\%) + \$300,000 \times (\text{Future Value of an 1, 6 periods, } 2.5\%) \\ & = \$200,000 \times 1.06152 \times 1.12649 \times 1.15969 + \$250,000 \times 1.12649 \times 1.15969 + \$300,000 \times 1.15969 \\ & = \underline{\$951,851} \end{aligned}$$

#### 12. 設第三期後每期顧客須付款 X 元

$$\begin{aligned} & \$20,000 + \$10,000 \times (\text{Present Value of an Ordinary Annuity, 2 periods, } 6\%) \\ & + X \times (\text{Present Value of an Ordinary Annuity, 6 periods, } 6\%) \times (\text{Present Value of an 1, 2} \\ & \text{periods, } 6\%) \\ & = \$300,000 \\ & \rightarrow \$20,000 + \$10,000 \times 1.83339 + X \times 4.91732 \times 0.89000 = \$300,000 \\ & \rightarrow X = \underline{\$59,790} \end{aligned}$$

流動資產包括作為正常營業週期之一部分而出售、消耗或實現之資產（例如存貨及應收帳款），即使不預期於報導期間後十二個月內實現亦然。應收分期帳款為企業預期於正常營業週期中收回變現之款項，符合流動資產之定義，故應全數於財務狀況表列為流動資產，唯須以現值表達