

習題答案

第一章

習題 1-1

1. 略 2. 略 3. 略 4. 略

$$5. (1) y = \left(x^2 + \frac{3}{e}\right)e^{-x} \quad (2) y = (x^2 + 2)e^{-x}$$

6. 略

習題 1-2

$$1. \frac{1}{2} \ln(1+x^2) = \ln|y| + c$$

$$3. x + \ln|x-1| + y - \ln|y+1| = c$$

$$5. \tan^{-1} x - \tan^{-1} y = c$$

$$7. x(y^2+1) = cy (c \neq 0)$$

$$9. y = 2 + ce^{-1/x}$$

$$11. x + y = \tan(x+c)$$

$$13. y = \frac{1}{2}(x+c)^2$$

$$15. \frac{1}{2}e^{x^2} + \frac{1}{6}y^6 - y = \frac{1}{2}$$

$$17. \frac{1}{2} \ln(x^2+1) + \ln|\sin y| = \frac{1}{2} \ln 2$$

$$19. \frac{y^2}{2} + 2y + \cos x = 1$$

$$21. e^y + ye^{-y} + e^{-y} = 4 - 2 \cos x$$

$$23. (x^2+1) \sin^2 y = 2$$

$$2. \frac{y^5}{5} + y = \frac{x^2}{2} + x + c$$

$$4. y^3 + e^y = \sin x + c$$

$$6. (\sin u + 1)(e^v + 1) = c (c \neq 0)$$

$$8. (x^2-1)^3 = cx^4y^2 (c \neq 0)$$

$$10. \frac{x^2}{2} + \frac{y^2}{2} + x - y + \ln|(x-1)(y+1)| = c$$

$$12. y - \tan \frac{x+y}{2} = c$$

$$14. \frac{x^3}{3} + x + \ln|y| = -\frac{4}{3}$$

$$16. \ln(1+x^2) - \ln|1+y| = \ln 5$$

$$18. \ln(1+e^x) - \ln|\cos y| = \frac{3}{2} \ln 2$$

$$20. y^4 - 2y^2 + 4 \ln|y| = x^4 - 4x - 1$$

$$22. |y|^3 = 4(1+x^3)$$

習題 1-3

1. (1) 零次

(2) 2 次

(3) 2 次

(4) $-\frac{2}{3}$ 次

(5) 零次

2. 略

3. $x^2 + y^2 = cx (c \neq 0)$

4. $\ln y - 2\sqrt{\frac{x}{y}}$

5. $\ln |y| + \frac{1}{6} \ln \left(\frac{3x^4}{y^4} + 1 \right) = c$

6. $\sin^{-1} \left(\frac{y}{x} \right) = \ln |x| + c$

7. $\frac{y}{1 + e^{\left(\frac{x}{y}\right)^2}} = c (c \neq 0)$

8. $\frac{x}{x-y} = \frac{1}{2} \ln |x| + c$

9. $y \left(2e^{\frac{x}{y}} + \frac{x}{y} \right) = c (c \neq 0)$

10. $\frac{y^2}{2} = x^2 (\ln |x| + c)$

11. $\frac{t}{x} + \ln |t| = c$

12. $\sin^{-1} \left(\frac{y}{x} \right) - \ln |x| = c$

13. $x^2 + y^2 = cy (c \neq 0)$

14. $x^8 = c(x^4 + y^4) (c > 0)$

15. $e^{-y/x} + \ln |x| = \frac{1}{e}$

習題 1-4

1. $(4y - x - 3)(2x + y - 3)^2 = c (c \neq 0)$

2. $\frac{3}{2}(x+y) + \ln |x+y-2| = x+c$

3. $x = cye^{xy} (c \neq 0)$

4. $\ln |y| - \frac{1}{2x^2y^2} - \frac{1}{xy} = c$

5. $x^2 - xy + y^2 + x - y = c (c > -\frac{1}{3})$

6. $5x^2 + 4xy + y^2 + 2x + 2y = c (c > -2)$

7. $(x^2 - y^2 - 1)^5 = c(x^2 + y^2 - 3) (c \neq 0)$

8. $(4x + 2y + 1)^4 = ce^{6x+2y} (c > 0)$

9. $y^2 - 3x^2 + 2xy + 18x + 2y = c$

10. $(4y - x - 3)(y + 2x - 3)^2 = c (c \neq 0)$

11. $x + 2y + \ln |2x + 3y - 1| = 2$

12. $3x^2 - 2xy - y^2 - 12x - 4y = 32$

習題 1-5

1. $N(x, y) = \frac{2}{xy^3} - \frac{3 \ln |x|}{y^4} + g(y)$

2. $M(x, y) = ye^{xy} + y^2 - \frac{y}{x^2} + h(x)$

3. $x^2y - y = c$

4. $x^3 + 2x^2y + y^2 = c$

5. $x^2 + e^{xy} = c$

6. $xe^{2y} - \sin xy + y^2 = c$

7. $y \sin x + x \cos y = c$

8. $ye^{x^2} - x^2 = c$

9. $\frac{1}{2} \ln(x^2 + y^2) + \tan^{-1} \frac{x}{y} = c$
10. $xy^2 + e^x \sin y = c$
11. $x^2 + \frac{x}{y} + \ln |y| = c$
12. $x^2 \sin \frac{y}{x} = c$
13. $xy = \left[\frac{-1}{c(n-1)} \right]^{\frac{1}{n-1}}$
14. $\frac{s(s-1)}{t} = c$
15. $2x + e^{xy} - y^2 = c$
16. $y \sin^2 x - y^2 \cos x = -9$
17. $\frac{y-3}{x} + \frac{2}{y} = 2$
18. 略
19. 略
20. 略
21. $\frac{1}{y} - \frac{x^2}{y^3} = c$
22. $3x^4 + 4x^3 + 6x^2y^2 = c$
23. $x = ce^{y^2}$
24. $ye^{x^3} - \frac{1}{3}e^{x^3} = c$
25. $xy^{\frac{3}{2}}e^{-\frac{1}{y}} = c$
26. $xy^{2/3}e^{y^{3/9}} = c$
27. $\frac{1}{x^2y^2} + \frac{1}{3x^3y^3} + \ln |y| = c$
28. $\frac{1}{2}x^2y^2 - \ln |y| = c$
29. $\frac{x^2}{2} + \tan^{-1} \frac{y}{x} = c$
30. $\sin^{-1}(xy) + \ln |x| = c$
31. $(2x^2y^2 + 4xy + y^4)e^{x^2} = c$
32. $\ln |xy| + y = c$
33. $y - \ln |x| - \frac{1}{xy} = c$
34. $\ln |x| - \frac{2x}{y} = c$
35. $x^4y - x^3y^2 = c$

習題 1-6

1. $y = \frac{c}{x^4} + \frac{x^5}{9}$
2. $y = ce^{-kx} + 100k$
3. $y = \frac{1}{2}(\sin x - \cos x) + ce^{-x}$
4. $z = ce^{\frac{x^2}{2}} + 1$
5. $y = x^2 \csc x + c \csc x$
6. $y = -\frac{1}{2} + ce^x$
7. $x = \frac{1}{3} + ce^{3/y}$
8. $y = \frac{1}{4} + \frac{c}{(x^2+1)^2}$
9. $y = (x+c) \cos x$
10. $(1 + \sin x)y = \frac{1}{4} \sin 2x + \frac{x}{2} + c$
11. $y = -2 \cos^2 x + c \cos x$
12. $x = \frac{1}{2} \ln y + \frac{c}{\ln y}$
13. $x \ln y = \frac{1}{2}(\ln y)^2 + c$
14. $x^2y^2 = x^3 - 3x^2 + c$

15. $\sin y = \frac{x^3 - 3x + c}{3(x+1)}$

16. $y = 1 - 5e^{\frac{x^2}{2}}$

17. $Q(t) = \frac{2(t^2 + 10t + 326)}{t + 5}, t > -5$

18. $y = (x-1)e^{3x} + 3e^{2x}$

19. $y = x^4 - 2x^2$ 20. $y = \frac{1}{3} + \frac{5}{3}e^{-x^3}$ 21. $r = \cos \theta \sin \theta + \frac{\sqrt{2}}{2} \cos \theta$

22. $y = \frac{e^x}{x}$

23. 略

24. (1) $y(y+2) = 1 + ce^{-x^2}$ (2) $y^2 = e^{-x^2}(c - 2\cos x)$ (3) $\tan y = x - 1 + ce^{-x}$

25. $y = 3e^t - 2(t+1)$

習題 1-7

1. $\frac{1}{y} = e^x(c-x)$

2. $y^{2/3} = cx^2 + \frac{2}{9}x^5$

3. $y^2 = \frac{1}{x^2(c-6x)}$

4. $y = \frac{1}{1 + ce^{x^{3/2}}}$

5. $\frac{1}{y^3} = x^3(2x^3 + c)$

6. $\frac{1}{y^2} = x + \frac{1}{2} + ce^{2x}$

7. $\frac{x^2}{y^2} = -\frac{2}{3}x^3\left(\frac{2}{3} + \ln x\right) + c$

8. $x^2 = 2 + \frac{c}{te^t}$

9. $y = \frac{1}{x^2\left(\frac{3}{2}\ln|x| + c\right)}$

10. $\frac{1}{y-1} = x + ce^x$

11. $c\left|\frac{x-1}{x+1}\right|^{1/2} = e^{\frac{1}{y-x}}$

12. $\frac{1}{y-1} = x - 1 + ce^{-x}$

13. $\frac{x}{y-x} = -\frac{1}{2} + ce^{4x}$

14. $x = \frac{1}{2 - y^2 + ce^{-\frac{1}{2}y^2}}$

15. $y^3 = \frac{1}{ce^x - 2x - 1}$

16. $x^2y = 4$

17. $y^4 = x^2 + \frac{15}{x^2}$

習題 1-8

1. $x + y = c$

2. $x^2 + y^2 = kx (k \neq 0)$

3. $xy = k (k \neq 0)$
5. $x - \frac{x^2}{2} - xy + \frac{y^2}{2} + y = k$ (k 為常數)
7. $x^2 - y^2 = k$ (k 為常數) 8. 約 2.71 小時
10. $N(t) = N_0 e^{-\frac{\ln 2}{20}t}$
12. 約 95.1 磅
14. (1) $\frac{dv}{dt} = -g - \frac{k}{m}v$
- (3) $t = -\frac{m}{k} \ln \left(1 + \frac{v_0 k}{mg} \right)$
15. $I(t) = \frac{1}{10} (1 - e^{-50t})$
17. (1) $I(t) = 5(1 - e^{-3t})$ (2) 約 4.75 安培
18. $i(t) = \frac{5}{101} (\sin 30t - 10 \cos 30t) + \frac{50}{101} e^{-3t}$
4. $2x^2 + y^2 = 2c$
6. $\frac{1}{2} \ln(x^2 + y^2) + \tan^{-1} \frac{y}{x} = k$ (k 為常數)
9. 略
11. (1) 約 39.6 分 (2) 約 70.5°C
13. $s(t) = 20 + 40e^{-\frac{t}{40}}$
- (2) $v(t) = \left(v_0 + \frac{mg}{k} \right) e^{-\frac{k}{m}t} - \frac{mg}{k}$
16. $I(t) = \frac{E}{R} (1 - e^{-\frac{R}{L}t})$
- (3) 5 安培
19. $Q(t) = 3(1 - e^{-4t}), I(t) = 12e^{-4t}$

第二章

習題 2-1

1. (1) 線性獨立 (2) 線性獨立
(3) 線性相依 (4) 線性相依
2. (1) 略 (2) $y = c_1 e^{2x} + c_2 e^{3x}$ (3) $y = 3e^{2x} - e^{3x}$
3. (1) 略 (2) $y = c_1 x^2 + \frac{c_2}{x^2}$ (3) $y = \frac{x^2}{4} + \frac{8}{x^2}$
4. $y_2 = -\frac{e^{-x}}{2x}$ 5. $y_2 = -\frac{\cos x}{x}$
6. $y_2 = -x - 1 (x \geq -\frac{1}{2})$, 通解為 $y = c_1 e^{2x} + c_2 (x + 1)$
7. $y_2 = x$ 8. $f(x) = x^3 + 3x + 1$
9. $y_2 = x^2 \ln x (x > 0)$, 通解為 $y = c_1 x^2 + c_2 x^2 \ln x$
10. $y = x^2 + c_1 x \ln |x| + c_2 x$

習題 2-2

1. $y = e^{-2x} (c_1 e^{2\sqrt{2}x} + c_2 e^{-2\sqrt{2}x})$
2. $y = c_1 e^{7x} + c_2 e^{-6x}$
3. $y = c_1 \cos 3x + c_2 \sin 3x$
4. $y = e^{-\frac{\sqrt{3}}{2}x} \left(c_1 \cos \frac{5}{2}x + c_2 \sin \frac{5}{2}x \right)$
5. $y = e^{2x} (c_1 \cos x + c_2 \sin x)$
6. $y = e^{\frac{3}{4}x} \left(c_1 \cos \frac{\sqrt{23}}{4}x + c_2 \sin \frac{\sqrt{23}}{4}x \right)$
7. $y = c_1 e^x + c_2 \cos x + c_3 \sin x$
8. $y = e^{-2x} (c_1 + c_2 x)$
9. $y = (c_1 + c_2 x) e^x + c_3 e^{3x}$
10. $y = c_1 e^x + e^{-x} (c_2 \cos x + c_3 \sin x)$
11. $y = c_1 e^{3x} + c_2 e^{-3x} + c_3 \cos \sqrt{2}x + c_4 \sin \sqrt{2}x$
12. $y = (c_1 + c_2 x + c_3 x^2) e^{2x}$
13. $y = c_1 + c_2 x + e^{\frac{x}{2}} \left(c_3 \cos \frac{\sqrt{3}}{2}x + c_4 \sin \frac{\sqrt{3}}{2}x \right)$
14. $y = (c_1 + c_2 x) \cos x + (c_3 + c_4 x) \sin x$
15. $y = e^{2x} + \cos 2x - \sin 2x$
16. $y = e^x - 2e^{2x} + e^{3x}$
17. $y = e^x (4 \cos 3x - \sin 3x)$

習題 2-3

1. (1) 略 (2) $y_c = c_1 e^x + c_2 e^{2x}$ (3) 略 (4) $y = c_1 e^x + c_2 e^{2x} + 2x^2 + 6x + 7$
2. $y = c_1 e^x + c_2 e^{2x} + \frac{1}{2} e^{3x} + 2x + 3$
3. $y = c_1 \cos x + c_2 \sin x + 3x + \frac{1}{2} x \sin x$
4. $y = c_1 \cos 2x + c_2 \sin 2x - 2x \cos 2x$
5. $y = c_1 e^x + c_2 e^{-2x} - \frac{1}{10} \cos 2x - \frac{3}{10} \sin 2x$
6. $y = e^{\frac{x}{2}} \left(c_1 \cos \frac{\sqrt{7}}{2}x + c_2 \sin \frac{\sqrt{7}}{2}x \right) + e^{2x} \left(\frac{5}{32} - \frac{3}{8}x + \frac{1}{4}x^2 \right)$
7. $y = e^{-2x} (c_1 + c_2 x) + \frac{1}{2} x^3 e^{-2x}$
8. $y = e^{2x} (c_1 + c_2 x) + \frac{1}{2} x^2 e^{2x} + e^x + \frac{1}{4}$
9. $y = c_1 e^{2x} + c_2 e^{-4x} + e^x \left(\frac{1}{13} \sin x - \frac{3}{26} \cos x \right)$
10. $y = e^{2x} (c_1 + c_2 x) + c_3 e^{-3x} + \frac{7}{10} x^2 e^{2x}$

$$11. y = c_1 + c_2 \cos x + c_3 \sin x + \ln |\csc x - \cot x| + \cos x \ln |\csc x| - x \sin x$$

$$12. y = c_1 + c_2 e^{2x} - \frac{1}{2} e^x \sin x$$

$$13. y = c_1 \cos 3x + c_2 \sin 3x - \frac{1}{12} x \cos 3x + \frac{1}{36} \sin 3x \ln |\sin 3x|$$

$$14. y = c_1 e^x + c_2 e^{2x} + c_3 e^{3x} + e^x \left(\frac{x}{2} + \frac{3}{4} \right)$$

$$15. y = e^{-x} (c_1 + c_2 x) + x^2 e^{-x} \left(\frac{1}{2} \ln x - \frac{3}{4} \right)$$

$$16. y = c_1 \cos x + c_2 \sin x - \cos x \ln |\sec x + \tan x|$$

$$17. y = c_1 + c_2 \cos x + c_3 \sin x + \ln |\sec x| - \sin x \ln |\sec x + \tan x|$$

$$18. y = e^x (c_1 + c_2 x) + x e^x (\ln |x| - 1)$$

習題 2-4

$$1. y_p = x^2 + 3x \quad 2. y_p = 2x^2 - 1 \quad 3. y_p = -\sin x - 3 \cos x \quad 4. y_p = \frac{e^{2x}}{2}$$

$$5. y_p = \frac{1}{9} x^3 + \frac{4}{9} x^2 + \frac{26}{27} x \quad 6. y_p = x(\sin x - \cos x)$$

$$7. y_p = -\frac{e^x}{73} (8 \cos 2x - 3 \sin 2x)$$

$$8. y_p = \frac{1}{15} (2 \cos 2x - \sin 2x) - \frac{1}{80} (3 \sin 3x + \cos 3x)$$

$$9. y_p = \frac{1}{10} x e^{3x} \quad 10. y_p = -e^{-3x} \left(\frac{x}{28} + \frac{39}{784} \right)$$

$$11. y_p = \frac{1}{2} e^{3x} (x^2 - x) - \frac{3}{8} e^x (\cos 2x + \sin 2x)$$

$$12. y_p = \frac{1}{2} x^2 e^{2x} + 2x e^x - \frac{1}{6} e^{-x}$$

習題 2-5

$$1. y = c_1 x^{1/2} + c_2 x^3 \quad 2. y = \frac{c_1}{x^2} + c_2 x^2 \quad 3. y = \frac{c_1}{x} + c_2 x + c_3 x^2$$

$$4. y = x^{-2} [c_1 \cos(3 \ln x) + c_2 \sin(3 \ln x)] \quad 5. y = c_1 + x^2 (c_2 + c_3 \ln x)$$

$$6. y(x) = \frac{4}{x} - \frac{3}{x^2} \quad 7. y = \frac{c_1}{2x+1} + c_2 (2x+1)^3$$

習題 2-6

1. 頻率 $f = \frac{7\sqrt{10}}{2\pi}$ (赫茲), 週期 $T = \frac{2\pi}{7\sqrt{10}}$ (秒)
2. $y(t) = -\frac{1}{2}e^{-6t}(1+6t)$ 呎, 圖形略
3. (1) $y(t) = \frac{1}{6}\cos 8t + \frac{1}{4}\sin 8t$ (2) $y(t) = -\frac{1}{6}\cos 8t + \frac{1}{4}\sin 8t$
4. 略 5. 圖形略, $y(t) = \cos 4t$, 頻率 $f = \frac{2}{\pi}$
6. $y = \frac{1}{6}\cos 4\sqrt{6}t$, 週期為 $\frac{\sqrt{6}\pi}{12}$ 秒, 頻率為 $\frac{2\sqrt{6}}{\pi}$ 週/秒, 振幅 $\frac{1}{6}$ 呎
7. 圖形略, $y(t) = e^{-t}(1+t)$ 8. $Y(t) = \frac{1}{15}(\cos t - \sin t)$
9. 全態電流 $I = e^{-2t}(c_1 \cos 4t + c_2 \sin 4t) + \cos 4t + 4 \sin 4t$
穩態電流 $I_p = \cos 4t + 4 \sin 4t$
10. $Q(t) = \frac{4}{697} \left[\frac{e^{-20t}}{3} (-63 \cos 15t - 116 \sin 15t) + 21 \cos 10t + 16 \sin 10t \right]$
 $I(t) = \frac{1}{2091} \left[e^{-20t} (-1920 \cos 15t + 13060 \sin 15t) + 120 (-21 \sin 10t + 16 \cos 10t) \right]$
11. $Q(t) = \cos t + \frac{1}{3} \sin t - \frac{1}{6} \sin 2t$
12. $Q(t) = Q_0 \cos \frac{t}{\sqrt{LC}} + \frac{CE_0}{1 - \omega^2 LC} \left(\sin \omega t - \sqrt{LC} \omega \sin \frac{t}{\sqrt{LC}} \right)$

第三章

習題 3-1

1. (1) $\frac{e^{-\pi s} + 1}{s^2 + 1}$ (2) $\frac{5(1 - e^{-3s})}{s}$ (3) $\frac{e^{2(1-s)} - 1}{1-s} + \frac{3}{s} e^{-2s}, s > 0$
(4) $\frac{3 - 4e^{-2s} + e^{-4s}}{s}$ (5) $\frac{s \sin b + a \cos b}{s^2 + a^2}, s > 0$
(6) $\frac{2}{s^3}, s > 0$ (7) $\frac{s \cos b - a \sin b}{s^2 + a^2}, s > 0$
2. (1) $\frac{1 - 2e^{-s}}{s^2}, s > 0$ (2) $\frac{2e^{-4s} - 1}{s}, s > 0$

3. 略 4. (1) $-2\sqrt{\pi}$ (2) $\frac{4\sqrt{\pi}}{3}$ (3) $-\frac{8\sqrt{\pi}}{15}$

5. (1) 略 (2) $\sqrt{\frac{\pi}{s}}, s > 0$ (3) $\frac{\sqrt{\pi}}{2}s^{-3/2}$

6. (1) $\frac{6+3s^2-2s^3}{s^4}$ (2) $\frac{4-3s+4s^2}{s^3}$ (3) $\frac{3s+1-2\ln 10}{(s-\ln 10)(s+1)}$

(4) $\frac{4(54+3s^2-4s^4)}{s^5}$ (5) $\frac{8}{s(s^2+16)}$ (6) $\frac{10(s+1)}{s^2+100}$ (7) $\frac{1}{s^2+4}$

(8) $\frac{s^2+2}{s(s^2+4)}$ (9) $\frac{2}{(s-3)^3}$ (10) $\frac{s(s^2+5)}{(s^2+9)(s^2+1)}$ (11) $\frac{3(s-8)}{s^2+4s+40}$

(12) $\frac{5}{(s+2)^2+25}$ (13) $\frac{\sqrt{\pi}}{2}(s+4)^{-3/2}$ (14) $\frac{s-\ln 2}{(s-\ln 2)^2+9}$

(15) $\frac{\sqrt{\pi}}{2s^{3/2}}e^{-1/4s}$ (16) $\sqrt{\frac{\pi}{s-2}}$

7. $\frac{2}{s^3} - \frac{120}{s^7} + \dots$

8. (1) $\cos \sqrt{6}t$ (2) $3e^{3t/2}$ (3) $2 \cos 3t - 6 \sin 3t$ (4) $2 \cos t + 3 \sin t$

(5) $\frac{1}{2}t^3 e^t + \frac{5}{24}t^4 e^t$ (6) $\frac{1}{2} \cos \sqrt{5}t + \frac{3\sqrt{5}}{20} \sin \sqrt{5}t$

(7) $2e^{2t}(3 \cos 4t + \sin 4t)$ (8) $e^t \cos \sqrt{2}t$ (9) $te^t - e^t + 1$

(10) $\frac{1}{4}e^{-t/2} + \frac{5}{8}te^{-t/2}$ (11) $\frac{e^{3t}}{\sqrt{\pi t}}$ (12) $e^{-t} + t - 1$

(13) $e^{2t} \cos 3t + \frac{2}{3}e^{2t} \sin 3t$

9. (1) $\frac{11}{3}e^{2t} - e^t + \frac{1}{3}e^{-t}$ (2) $\frac{5}{3}e^{2t} - \frac{2}{3}e^{-t}$ (3) $-\frac{3}{4} + \frac{3}{4}e^{-2t} + \frac{5}{2}te^{-2t}$

(4) $\frac{1}{4} - \frac{1}{4} \cos 2t + \frac{1}{2} \sin 2t$ (5) $\frac{1}{2}t^2 e^{-t} - \frac{1}{6}t^3 e^{-t}$ (6) $1 + 4t - e^{-t}$

(7) $\frac{1}{2} + \frac{1}{4}e^{2t} + \frac{1}{4}e^{-2t}$ (8) $(3t-5)e^{2t} + (2t+5)e^t$

(9) $\frac{1}{16}e^{-3t} - \frac{1}{16}e^t + \frac{1}{4}te^t$ (10) $\frac{1}{2}(e^{-2t} - \sin 2t)$ (11) $e^{2t} + 4te^{2t} + 2t^2 e^{2t}$

習題 3-2

1. (1) $\frac{2}{(s-1)^3}$ (2) $\frac{s(s+2)}{(s^2+2s+2)^2}$ (3) $\frac{2a(3s^2-a^2)}{(s^2+a^2)^3}$ (4) $\frac{s^2-7}{(s^2+7)^2}$
- (5) $\frac{6}{(s-4)^4}$ (6) $\frac{2s(s^2-3a^2)}{(s^2+a^2)^3}$ (7) $\frac{18(s^2+2s-2)}{(s^2+2s+10)^3}$ (8) $\frac{105}{16}\sqrt{\pi}s^{-9/2}$
- (9) $\frac{\pi}{2}-\tan^{-1}\frac{s}{3}$ (10) $\ln\left|\frac{s+1}{s}\right|$
2. (1) te^{2t} (2) $\cos t - \frac{1}{2}t \sin t$ (3) $\frac{1}{2}(1-\cos 2t)$ (4) $\frac{1}{2}t \sinh 2t$
- (5) $\frac{e^{-bt}-e^{-at}}{t}$ (6) $\frac{e^t \sin t}{t}$ (7) $\frac{1-e^{-t}}{t}$

習題 3-3

1. $y=(1+t)e^{-2t}$ 2. $y=\frac{1}{2}(e^{-x}-\cos x+\sin x)$ 3. $2 \cos 2x+\sin 2x$
4. $I=0.1-0.1e^{-50t}$ 5. $y=\cos t+t \sin t$ 6. $y=5e^{2t}-2t^2$
7. $y=x+\cos x-3 \sin x$ 8. $y=\frac{7}{24}e^{-t} \sin 3t+\frac{1}{8}e^{-t} \sin t$
9. $y=-1+\frac{1}{2}e^x+\frac{1}{2}\cos x-\frac{1}{2}\sin x$
10. $Q=2(3-3e^{-4t}\cos 3t-4e^{-4t}\sin 3t)$ 11. $x=2e^{-2t}-e^{-3t}, y=2(e^{-3t}-e^{-2t})$
12. $y=te^t\left(\frac{t^3}{24}+\frac{t}{2}-1\right)$
13. $y=-\frac{1}{4}+\frac{7}{8}e^{2x}+\frac{3}{8}e^{-2x}, z=x-\frac{7}{4}e^{2x}+\frac{3}{4}e^{-2x}$ 14. $x=\sin t, y=\cos t$
15. $x=\frac{1}{2}(e^{3t}+e^t), y=\frac{1}{2}(e^{3t}-e^t), z=\frac{1}{2}(e^{3t}-e^t)$
16. $x=\frac{1}{10}(e^t-e^{-t}+te^{-t}), y=\frac{1}{20}(e^t-e^{-t}-4te^{-t})$
17. $w=e^x, y=e^x+e^{-x}, z=2e^{-x}$ 18. $x=e^t+e^{2t}, y=e^{2t}$
19. $x=5e^{-t}+3e^{4t}, y=5e^{-t}-2e^{4t}$

習題 3-4

$$1. (1) \frac{2e^{-s}}{s^3} \quad (2) \frac{1-e^{-2s}}{s} \quad (3) \frac{1}{s^2+1} + e^{-\pi s} \left(\frac{\pi}{s} + \frac{1}{s^2} + \frac{1}{s^2+1} \right)$$

$$(4) \frac{s(1+e^{-\pi s})}{s^2+1} \quad (5) \frac{3e^{-s}}{s} \left(1 + \frac{3}{s} + \frac{4}{s^2} + \frac{4}{s^3} \right)$$

$$(6) \frac{2e^{-2s}}{s} \left(2 + \frac{2}{s} + \frac{1}{s^2} \right)$$

$$2. (1) \frac{1}{2} t^2 - u(t-2)(t-2)^2 + \frac{1}{2} u(t-4)(t-4)^2 \quad (2) \frac{1}{2} u(t-4)(t-4)^2$$

$$(3) u(t-2) \cos 4(t-2) \quad (4) u\left(t-\frac{\pi}{3}\right) \sin\left(t-\frac{\pi}{3}\right)$$

$$(5) u(t-3)e^{-3(t-3)} \sin(t-3)$$

$$3. y=2u(x-1)[1-\cos(x-1)] \quad 4. y=-1+\cos 4t+u(t-3)[1-\cos 4(t-3)]$$

習題 3-5

$$1. \frac{1-e^{-s}(s+1)}{s^2(1-e^{-s})} \quad 2. \frac{1}{s(1-e^{-s})} \quad 3. \frac{e^{-2s}(s+1)+s-1}{s^2(1-e^{-2s})}$$

$$4. \frac{1-e^{-s}-se^{-2s}}{s^2(1-e^{-2s})} \quad 5. \frac{\omega}{s^2+\omega^2} \coth \frac{\pi s}{2\omega} \quad 6. \frac{1}{s^2} \tanh \frac{s}{2}$$

$$7. \frac{2(1-e^{-s/2})}{s(s^2+4\pi^2)(1+e^{-s/2})} - \frac{2}{s^2+4\pi^2} \quad 8. \frac{\omega}{(s^2+\omega^2)(1-e^{-\pi s/\omega})}$$

習題 3-6

$$1. \frac{1}{2} t \sin t \quad 2. e^{3t} - e^{2t} \quad 3. \frac{t^{n-1}}{(n-1)!}$$

$$4. (1) \frac{1}{4} (1-\cos 2t) \quad (2) t-\sin t \quad (3) te^{-t}+2e^{-t}+t-2 \quad (4) \frac{1}{2} t \sin t$$

$$(5) \frac{1}{18} e^{-2t} \left(\frac{1}{3} \sin 3t - t \cos 3t \right) \quad (6) \frac{1}{2\omega} t \sin \omega t$$

$$5. y(t)=\sin t \quad 6. y(t)=2e^t \quad 7. y(t)=1 \quad 8. y(t)=t^2+\frac{t^4}{12}$$

$$9. y(t)=1+\frac{t^2}{2} \quad 10. y(t)=2 \quad 11. \text{略}$$

習題 3-7

$$1. \begin{cases} v_1(t) = e^{-t} \cos t \\ v_2(t) = e^{-t} (\cos t + \sin t) \end{cases}$$

$$2. y(t) = 3u(t-2)e^{-(t-2)}$$

$$3. y(t) = u(t-1) \sin(t-1)$$

第四章

習題 4-1

$$1. y = c_0 x + 2(x-1) + \frac{1}{2} (x-1)^2 - \frac{1}{6} (x-1)^3 + \frac{1}{12} (x-1)^4 - \cdots$$

$$2. y = c_0 \left[1 + x + \frac{1}{2!} (1+c_0)x^2 + \frac{1}{3!} (1+5c_0+2c_0^2)x^3 + \frac{1}{4!} (1+17c_0+26c_0^2+6c_0^3)x^4 + \cdots \right]$$

$$3. y = x + \frac{1}{2} x^2 + \frac{2}{3} x^3 + \frac{1}{3} x^4 + \cdots$$

$$4. y = c_0 \left(1 - \frac{x^2}{2} + \frac{x^4}{24} + \cdots \right) + c_1 \left(x - \frac{x^3}{6} + \frac{x^5}{120} + \cdots \right)$$

$$5. y = c_0 \left[1 + \frac{1}{2} x^2 - \sum_{k=2}^{\infty} (-1)^k \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2k-3)}{2^k k!} x^{2k} \right] + c_1 x$$

$$6. y = c_0 \left(1 + \frac{x^3}{6} + \frac{x^6}{180} + \cdots \right) + c_1 \left(x + \frac{x^4}{12} + \frac{x^7}{504} + \cdots \right)$$

$$7. y = c_0 \left[1 - \frac{1}{2} (x-2)^2 + \frac{1}{6} (x-2)^3 - \frac{1}{12} (x-2)^4 + \frac{1}{20} (x-2)^5 - \cdots \right] \\ + c_1 \left[x-2 - \frac{1}{2} (x-2)^2 - \frac{1}{6} (x-2)^3 + \frac{1}{6} (x-2)^4 - \frac{1}{36} (x-2)^5 + \cdots \right]$$

$$8. y = c_0 \left(1 + \frac{x^2}{2} + \frac{x^4}{24} + \frac{x^5}{20} + \frac{x^6}{720} + \cdots \right) + c_1 \left(x + \frac{x^3}{6} + \frac{x^4}{12} + \frac{x^5}{120} + \frac{7x^6}{360} + \cdots \right)$$

$$9. y = c_0 \left(1 - \frac{2}{3} x^3 - \frac{2}{45} x^6 - \frac{2}{405} x^9 - \cdots \right) + c_1 \left(x - \frac{1}{6} x^4 - \frac{1}{63} x^7 - \frac{1}{567} x^{10} - \cdots \right) \\ + x^2 + \frac{1}{3} x^3 + \frac{1}{12} x^4 + \frac{1}{45} x^6 + \frac{1}{126} x^7 + \frac{1}{405} x^9 \cdots$$

$$10. y = \frac{x^2}{6} + \frac{x^3}{54} + \frac{x^4}{324} + \frac{4x^5}{1215} + \cdots$$

$$11. y = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{6} + \cdots$$

$$12. y = 1 - x + \frac{1}{2}(x-1)^2 - \frac{1}{6}(x-1)^3 + \frac{1}{6}(x-1)^4 + \cdots$$

習題 4-2

$$1. (1) 2 \quad (2) 0, -1 \quad (3) -1 \quad (4) -\frac{1}{2}$$

$$2. (1) r^2 + r = 0 \quad (2) r^2 - \frac{3}{2}r + \frac{1}{2} = 0 \quad (3) (r+1)^2 = 0$$

$$3. y = c_1 x^{-1}(1 + 3x + \cdots) + c_2 x^{1/2} \left(1 + \frac{3}{10}x + \frac{27}{280}x^2 + \cdots \right)$$

$$4. y = c_1 \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n+1)} + c_2 \sqrt{x} e^{-x/2}$$

$$5. y = c_1 \left(1 - \frac{x^3}{24} + \frac{x^6}{2448} - \cdots \right) + c_2 x^{1/3} \left(1 - \frac{x^3}{30} + \frac{x^6}{2420} - \cdots \right)$$

$$6. y = c_1 \sqrt{x} \left(1 - \frac{x^2}{6} + \frac{x^4}{168} - \frac{x^6}{11088} + \cdots \right) + c_2 x \left(1 - \frac{x^2}{10} + \frac{x^4}{360} - \frac{x^6}{28080} - \cdots \right)$$

$$7. y(x) = c_1 y_1(x) + c_2 y_2(x), \text{ 其中 } y_1 = x^{-1} \sum_{n=0}^{\infty} (-1)^n \frac{2n+3}{3n!} x^{-n}, y_2 = x^{-1/2} \left[1 + \sum_{n=1}^{\infty} \frac{(-1)^n 2^n (n+1)}{1 \cdot 3 \cdot 5 \cdots (2n-1)} x^{-n} \right]$$

$$8. y = c_1 x^{1/2} \left(1 - \frac{x^2}{6} + \cdots \right) + c_2 x^{-1/2} \left(1 - \frac{x^2}{2} + \cdots \right)$$

習題 4-3

$$1. \text{略} \quad 2. \text{略} \quad 3. \text{略} \quad 4. \text{略} \quad 5. \text{略}$$

$$6. (1) x^n J_n(x) + c \quad (2) -x^{-n} J_n(x) + c \quad 7. \text{略}$$

$$8. (1) x^4 J_2(x) - 2x^3 J_3(x) + c \text{ 或 } (8x^2 - x^4) J_0(x) + (4x^3 - 16x) J_1(x)$$

$$(2) -x^3 J_2(x) - 5x^2 J_1(x) - 15x J_0(x) + 15 \int J_0(x) dx$$

$$9. y = c_1 J_{1/3}(x^2) + c_2 J_{-1/3}(x^2)$$

習題 4-4

1. $P_5(x) = \frac{1}{8} (63x^5 - 70x^3 + 15x)$ 2. 略
3. $x^3 = \frac{3}{5} P_1(x) + \frac{2}{5} P_3(x)$ 4. $P_5(x) = \frac{1}{8} (63x^5 - 70x^3 + 15x)$
5. 略 6. 略 7. 略

第五章

習題 5-1

1. $x = -2, y = 3$ 2. $X = \frac{1}{4} \begin{bmatrix} -11 & -24 & -27 \\ -13 & -15 & 103 \end{bmatrix}$
3. (1) $A^2 = \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix}, A^3 = \begin{bmatrix} -7 & 30 \\ 60 & -67 \end{bmatrix}$ (2) $\begin{bmatrix} -13 & 52 \\ 104 & -117 \end{bmatrix}$ ₁
4. $A^2 = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}, A^3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
5. 略 6. $\begin{bmatrix} 1 & 2n-1 \\ 0 & 1 \end{bmatrix}$ 7. 不一定
8. $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A^3 = \begin{bmatrix} 0 & a \\ \frac{1}{a} & 0 \end{bmatrix}, A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A^5 = \begin{bmatrix} 0 & a \\ \frac{1}{a} & 0 \end{bmatrix}, A^6 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
9. 略 10. 略 11. 略
12. $AB = \begin{bmatrix} 15 & -5 & -10 \\ -5 & 21 & 6 \\ -10 & 6 & 11 \end{bmatrix}, BA = \begin{bmatrix} 21 & -9 & -2 & -7 \\ -9 & 10 & -3 & 0 \\ -2 & -3 & 2 & 3 \\ -7 & 0 & 3 & 6 \end{bmatrix}$
13. D 爲反對稱方陣
14. 略 15. 略 16. 略

習題 5-2

$$1. (1) A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ -\frac{1}{5} & \frac{3}{5} \end{bmatrix} \quad (2) B \text{ 爲不可逆} \quad (3) C^{-1} = \begin{bmatrix} -\frac{1}{11} & \frac{2}{11} \\ \frac{4}{11} & \frac{3}{11} \end{bmatrix}$$

$$2. \begin{bmatrix} \frac{2}{7} & 1 \\ \frac{1}{7} & \frac{3}{7} \end{bmatrix} \quad 3. = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \quad 4. \begin{bmatrix} -\frac{1}{4} & \frac{1}{4} \\ -\frac{3}{16} & \frac{1}{8} \end{bmatrix}$$

4. (1) 否 (2) 是

$$6. \text{略} \quad 7. x=2 \quad 8. \text{略} \quad 9. 0 \quad 10. \begin{bmatrix} -\frac{1}{12} & -\frac{1}{24} \\ \frac{5}{24} & \frac{1}{24} \end{bmatrix}$$

$$11. (A^T)^{-1} = (A^{-1})^T$$

習題 5-3

1. (2)(3)(5)

$$2. (1) \frac{1}{6}R_3 \quad (2) 7R_1 + R_2 \quad (3) \frac{1}{5}R_3 + R_1 \quad (4) R_1 \leftrightarrow R_3, R_3 \leftrightarrow R_4$$

$$3. (1) E_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad (2) E_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$(3) E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \quad (4) E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$4. A^{-1} = \begin{bmatrix} \frac{1}{6} & \frac{1}{2} & -\frac{5}{6} \\ -\frac{1}{6} & \frac{1}{2} & -\frac{2}{3} \\ \frac{1}{6} & -\frac{1}{2} & \frac{7}{6} \end{bmatrix} \quad 5. B^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

$$6. \mathbf{C}^{-1} = \begin{bmatrix} 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 1 & 0 & 0 & -1 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ -1 & 0 & 1 & 1 \end{bmatrix} \quad 7. \mathbf{D}^{-1} = \begin{bmatrix} -\frac{1}{2} & 1 & \frac{3}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & \frac{1}{2} \end{bmatrix}$$

8. 不可逆

9. 不可逆

10. $\mathbf{A}$ 與 $\mathbf{B}$

$$11. a=1, \mathbf{A}^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \quad 12. \mathbf{B} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

習題 5-4

$$1. \begin{cases} x_1 = 2t \\ x_2 = \frac{5}{3}t - \frac{1}{3}, t \in \mathbb{R} \\ x_3 = t \end{cases} \quad 2. x_1 = -\frac{3}{4}, x_2 = -\frac{5}{4}, x_3 = \frac{13}{4}$$

$$3. x_1 = 1, x_2 = 2, x_3 = 2 \quad 4. \begin{cases} x_1 = \frac{1}{2} + s \\ x_2 = 1 + 2s - t \\ x_3 = s \\ x_4 = t \end{cases}, s \in \mathbb{R}, t \in \mathbb{R}$$

$$5. x_1 = 1, x_2 = -1, x_3 = 2 \quad 6. x_1 = -\frac{3}{4}, x_2 = -\frac{5}{4}, x_3 = \frac{13}{4}$$

$$7. I_1 = \frac{9}{31} \text{ 安培}, I_2 = \frac{24}{31} \text{ 安培}, I_3 = -\frac{33}{31} \text{ 安培 (負表示與原來 } I_3 \text{ 方向相反)}$$

$$8. (1) a = -3 \quad (2) a \neq \pm 3 \quad (3) a = 3$$

$$9. (2) (3)$$

$$10. x_1 = \frac{3}{2}, x_2 = \frac{1}{2}, x_3 = \frac{3}{2} \quad 11. x_1 = 5, x_2 = 4, x_3 = 7$$

$$12. 3 \text{ 與 } -2 \quad 13. \begin{cases} x_1 = -\frac{t}{3} \\ x_2 = \frac{2}{3}t, t \in \mathbb{R} \\ x_3 = t \end{cases}$$

習題 5-5

1. (1) -66 (2) -40 (3) -240

2. (1) 0 (2) 0 (3) -78 (4) 2

3. $A_{13} = -9$, $A_{23} = 0$, $A_{33} = 3$, $A_{43} = -2$

4. 略 5. -4 、 -1 與 0 6. 略

7. $0, 3$ 與 -5

$$8. (1) A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -5 & 2 & 1 \\ \frac{7}{2} & -\frac{3}{2} & -\frac{1}{2} \end{bmatrix} \quad (2) B^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -1 & 1 & 0 \end{bmatrix} \quad (3) C^{-1} = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & -6 \\ -2 & -1 & 5 \end{bmatrix}$$

9. $x_1 = \frac{15}{34}$, $x_2 = -\frac{1}{34}$, $x_3 = -\frac{3}{17}$

10. $x_1 = 2$, $x_2 = 2$, $x_3 = -\frac{4}{3}$

11. $x = 1$, $y = -1$, $z = 2$

12. $x = \frac{35}{18}$, $y = \frac{29}{18}$, $z = \frac{5}{18}$

13. $0, 4$

14. 略 15. 略

16. $x_1 = 4$, $x_2 = 8$, $x_3 = 19$ 17. $x_1 = 3$, $x_2 = -2$, $x_3 = 1$, $x_4 = 2$

18. (1) 有非零解 (2) 無非零解 19. (1) 略 (2) 略

第六章

習題 6-1

1. $-\frac{15}{\sqrt{59}} \mathbf{i} - \frac{5}{\sqrt{59}} \mathbf{j} + \frac{35}{\sqrt{59}} \mathbf{k}$ 2. 略

3. $\cos \alpha = \frac{4}{\sqrt{53}}$, $\cos \beta = \frac{-1}{\sqrt{53}}$, $\cos \gamma = \frac{6}{\sqrt{53}}$ 4. $\left\langle \frac{7}{\sqrt{6}}, \frac{7}{\sqrt{3}}, \frac{7}{\sqrt{2}} \right\rangle$

5. $\left(\frac{4}{3}, 1, \frac{4}{3} \right)$ 6. $a = -2$, $b = 1$, $c = -3$ 7. $\begin{cases} x = 2 + 2t \\ y = -3 + 5t, t \in \mathbb{R} \\ z = 1 + 4t \end{cases}$

8. $\angle ABC \approx 0.2288$ 弧度 $\approx 13.11^\circ$

9. $\left\langle -\frac{60}{7}, \frac{20}{7}, \frac{30}{7} \right\rangle$ 與 $\left\langle \frac{60}{7}, -\frac{20}{7}, -\frac{30}{7} \right\rangle$

10. $\text{proj}_{\mathbf{v}} \mathbf{u} = \frac{5}{19} \mathbf{i} + \frac{15}{19} \mathbf{j} - \frac{15}{19} \mathbf{k}$, $\mathbf{u}$ 在 $\mathbf{v}$ 上之投影長度為 $\frac{5\sqrt{19}}{19}$

$$11. \left\langle \frac{6}{25}, \frac{12}{25}, -\frac{3\sqrt{5}}{25} \right\rangle + \left\langle -\frac{81}{25}, \frac{13}{25}, -\frac{22\sqrt{5}}{25} \right\rangle$$

$$12. \text{略} \quad 13. \frac{5\sqrt{6}}{6} \quad 14. 24 \text{ 呎-磅}$$

$$15. \frac{4}{9} \mathbf{i} + \frac{1}{9} \mathbf{j} + \frac{8}{9} \mathbf{k}, -\frac{4}{9} \mathbf{i} - \frac{1}{9} \mathbf{j} - \frac{8}{9} \mathbf{k}$$

$$16. 3\sqrt{5} \quad 17. \sqrt{638} \quad 18. \frac{3\sqrt{33}}{2} \quad 19. 11$$

$$20. (1) 9 \quad (2) \sqrt{35} \quad (3) \theta \approx 0.6982 \text{ 弧度} \approx 40.01^\circ$$

$$21. \text{略} \quad 22. \frac{88}{3} \quad 23. \frac{x+1}{6} = \frac{y-2}{-1} = \frac{z-3}{4}$$

$$24. 4x + 7y - 5z + 3 = 0 \quad 25. 3x - y - z = 2$$

$$26. \text{向量方程式 } \mathbf{x} = 2\mathbf{i} - \mathbf{j} + 6\mathbf{k} + t(\mathbf{i} + 2\mathbf{j} - 8\mathbf{k}), t \in \mathbb{R}$$

$$\text{參數方程式 } x = 2 + t, y = -1 + 2t, z = 6 - 8t, t \in \mathbb{R}$$

$$\text{對稱方程式 } \frac{x-2}{1} = \frac{y+1}{2} = \frac{z-6}{-8}$$

$$27. \text{略} \quad 28. \text{略}$$

習題 6-2

$$1. (1) \text{非向量空間} \quad (2) \text{非向量空間} \quad (3) \text{向量空間}$$

$$2. (2) (3) \quad 3. \text{略} \quad 4. \text{略} \quad 5. \text{略} \quad 6. \text{略} \quad 7. \text{略} \quad 8. \text{略}$$

$$9. (1) \text{略} \quad (2) \text{略} \quad 10. \text{否}$$

$$11. \langle -2, -1, 1 \rangle \text{ 為 } \mathbf{x}_1, \mathbf{x}_2 \text{ 與 } \mathbf{x}_3 \text{ 之線性組合}$$

$$12. \text{是} \quad 13. \text{不是} \quad 14. (2) (3)$$

$$15. \text{可以} \quad 16. \text{不可以}$$

$$17. (1) \text{為線性相依}, \langle 1, -2, 3 \rangle = \frac{1}{2} [\langle 4, 2, 1 \rangle - \langle 2, 6, -5 \rangle]$$

$$(2) \text{為線性相依}, \langle 3, 6, 6 \rangle = 2 \langle 1, 1, 0 \rangle + \langle 0, 2, 3 \rangle + \langle 1, 2, 3 \rangle$$

$$18. (3)$$

$$19. \text{解空間之基底為 } \left\{ \begin{bmatrix} 3 & -3 & -1 & 1 & 0 \end{bmatrix}^T, \begin{bmatrix} 1 & -1 & -\frac{1}{2} & 0 & 1 \end{bmatrix}^T \right\}, \text{二維}$$

$$20. \text{三維} \quad 21. \{0\}$$

習題 6-3

$$1. (1) \left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 0 \\ 7 \end{bmatrix} \right\} \quad (2) \left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$$2. (1) \left\{ \langle 1, 4, 5, 2 \rangle, \left\langle 0, 1, 1, \frac{4}{7} \right\rangle \right\} \quad (2) \left\{ \left\langle 1, 0, -\frac{1}{2} \right\rangle \right\}$$

$$3. (1) \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} \right\} \quad (2) \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -5 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 15 \\ 18 \end{bmatrix} \right\}$$

$$4. \left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\} \quad 5. \{ \langle 1, 4, 5, 6, 9 \rangle, \langle 3, -2, 1, 4, -1 \rangle \}$$

$$6. (1) \text{nullity}(\mathbf{A})=2 \quad (2) \text{nullity}(\mathbf{B})=2$$

習題 6-4

$$1. \frac{\pi}{2} \quad 2. 2 \quad 3. 7\sqrt{2} \quad 4. \frac{2\sqrt{6}}{3} \quad 5. 1 \quad 6. \text{略}$$

$$7. \text{略} \quad 8. \left\{ \left\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0 \right\rangle, \left\langle \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3} \right\rangle \right\}$$

$$9. \left\{ \langle 2, 1, 2 \rangle, \left\langle \frac{1}{9}, -\frac{4}{9}, \frac{1}{9} \right\rangle, \langle -2, 0, 2 \rangle \right\} \text{ 為正交基底}$$

$$\left\{ \left\langle \frac{2}{3}, \frac{1}{3}, \frac{2}{3} \right\rangle, \left\langle \frac{\sqrt{2}}{6}, -\frac{2\sqrt{2}}{3}, \frac{\sqrt{2}}{6} \right\rangle, \left\langle -\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2} \right\rangle \right\}$$

為正規正交基底

$$10. \left\{ \left\langle \frac{\sqrt{5}}{5}, \frac{2\sqrt{5}}{5}, 0 \right\rangle, \left\langle -\frac{3\sqrt{70}}{35}, \frac{3\sqrt{70}}{70}, \frac{\sqrt{70}}{14} \right\rangle \right\} \text{ 為正規正交基底}$$

$$11. \left\{ \left\langle \frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2} \right\rangle, \left\langle \frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{6} \right\rangle, \left\langle \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3} \right\rangle \right\} \text{ 為正規正交基底}$$

$$12. \left\{ 1, \sqrt{3}(2x-1), 6\sqrt{5} \left(x^2 - x + \frac{1}{6} \right) \right\} \text{ 為正規正交基底}$$

第七章

習題 7-1

1. (1) L 為線性變換2. (1) 表示在平面上以 y -軸為對稱軸，而將點 (x, y) 對應到點 $(-x, y)$ 上(2) 表示在平面上以原點為對稱點，將點 (x, y) 對應到點 $(-x, -y)$ 上(3) 表示在平面上，將點 (x, y) 對應到逆時鐘方向旋轉 90° 的點 $(-y, x)$ 上3. 略 4. $L(\langle x, y, z \rangle) = \langle 4x - 2y - z, 3x - 4y + z \rangle; \langle 9, 23 \rangle$

5. $\begin{bmatrix} 1 \\ -19 \end{bmatrix}$ 6. 略 7. (1) $\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

8. (1) $\begin{bmatrix} 0 & 0 & 2 \\ 3 & 1 & 0 \\ 2 & 0 & -1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

9. (1) $\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$ (2) (i) $\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$ (ii) $\begin{bmatrix} -15 \\ 9 \\ 6 \end{bmatrix}$

10. $\begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix}$ 11. $\begin{bmatrix} \frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} \\ \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} -\frac{3\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$

12. (1) $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & -\frac{1}{2} \\ 1 & \frac{3}{4} \end{bmatrix}$ (3) $\begin{bmatrix} 3 & 2 \\ -4 & 4 \end{bmatrix}$ (4) $\begin{bmatrix} -2 & 2 \\ \frac{1}{2} & 2 \end{bmatrix}$

(5) (i) $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$ (iii) $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$ (iv) $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$

13. (1) $\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 0 \\ -3 & 1 & 0 \end{bmatrix}$ (3) (i) $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

14. (1) $\begin{bmatrix} \frac{7}{3} & -\frac{4}{3} \\ -\frac{2}{3} & \frac{5}{3} \\ \frac{2}{3} & -\frac{2}{3} \end{bmatrix}$ (2) $\begin{bmatrix} -3 \\ 4 \\ 3 \end{bmatrix}$

$$15. (1) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 4 \\ 0 & 2 & 5 \\ 1 & 3 & 1 \end{bmatrix} \quad (2) -3x^2+5x^3-2x^4 \quad 16. (1) \{1\} \quad (2) \{t, 1\}$$

$$17. \{\langle 1, 0, 0 \rangle, \langle 0, 1, 0 \rangle, \langle 0, 0, 1 \rangle\}$$

習題 7-2

$$1. (1) P(\lambda)=\lambda^2-4 \quad (2) P(\lambda)=\lambda^2-5\lambda+7 \quad (3) P(\lambda)=\lambda^2+3$$

$$(4) P(\lambda)=\lambda^3-6\lambda^2+12\lambda-8 \quad (5) P(\lambda)=\lambda^3-4\lambda^2+7$$

$$2. (1) 1, 2 \text{ 與 } 4; \text{ 固有向量分別為 } \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ 與 } \begin{bmatrix} 7 \\ -4 \\ 2 \end{bmatrix}$$

$$(2) 2 \text{ 與 } 3; \text{ 固有向量分別為 } \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ 與 } \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$(3) 0, 2 \text{ 與 } 3; \text{ 固有向量分別為 } \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix} \text{ 與 } \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$3. \text{ 對應於 } \lambda=1 \text{ 的固有空間的基底為 } \left\{ \begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\};$$

$$\text{對應於 } \lambda=-2 \text{ 的固有空間的基底為 } \left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\};$$

$$\text{對應於 } \lambda=-1 \text{ 的固有空間的基底為 } \left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\};$$

$$4. 1, -512, 0, \frac{1}{512} \quad 5. (1) \text{ 略 } (2) \text{ 略}$$

習題 7-3

1. (2) (3) (4)

2.
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

3.
$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

4.
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

5.
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

6. (1) $J = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, J_1 = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}, J_2 = [2]$

(2) $J = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{bmatrix}, J_1 = [-2], J_2 = \begin{bmatrix} 4 & 1 \\ 0 & 4 \end{bmatrix}$

7.
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 32 & 32 \\ 0 & 32 & 32 \end{bmatrix}$$

8.
$$\begin{bmatrix} -1 & 10237 & -2047 \\ 0 & 1 & 0 \\ 0 & 10245 & -2048 \end{bmatrix}$$

9.
$$\frac{1}{6} \begin{bmatrix} 1 & -3 & 7 \\ -1 & 9 & -13 \\ 1 & 3 & -5 \end{bmatrix}$$

10. 略 11. (1)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \quad (2) \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}$$

12.
$$Q = \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{bmatrix}; Q^T A Q = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$$13. Q = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}; Q^T A Q = \begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix}$$

$$14. Q = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix}; Q^T A Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$15. Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}; Q^T A Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

習題 7-4

$$1. \begin{bmatrix} e^3 & 2e^2 - 2e^3 & -\frac{1}{3} + \frac{1}{3}e^3 \\ 0 & e^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad 2. \begin{bmatrix} -e^2 + 2e & -e^2 + e \\ 2e^2 - 2e & 2e^2 - e \end{bmatrix}$$

$$3. \begin{bmatrix} 2e^2 - e^3 & -2e + 2e^2 & \frac{e}{2} - \frac{e^3}{2} \\ -e^2 + e^3 & 2e - e^2 & -\frac{e}{2} + \frac{e^3}{2} \\ -4e^2 + 4e^3 & 4e - 4e^2 & -e + 2e^3 \end{bmatrix}$$

$$4. e^{-t} \begin{bmatrix} 1+t & 2t + \frac{t^2}{2} & t \\ 0 & 0 & 0 \\ -t & -t - \frac{t^2}{2} & -t+1 \end{bmatrix} \quad 5. \text{略} \quad 6. \text{略}$$

習題 7-5

1. (3) (4)

$$2. (1) \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$(2) \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 3 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$(3) \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & -5 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ -5 & 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$(4) \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 9 & 3 & -4 \\ 3 & -1 & \frac{1}{2} \\ -4 & \frac{1}{2} & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$3. (1) x_1^2 - 3x_2^2 + 5x_3^2 \quad (2) 2x_1^2 - 6x_1x_2 + 5x_2^2$$

$$(3) 2x_1x_2 + 2x_1x_3 + 2x_1x_4 + 2x_2x_3 + 2x_2x_4 + 2x_3x_4$$

$$4. \text{最大值 } 4, \pm \begin{bmatrix} \frac{\sqrt{6}}{6} \\ \frac{\sqrt{6}}{6} \\ \frac{\sqrt{6}}{3} \end{bmatrix}; \text{最小值 } -2, \pm \begin{bmatrix} -\frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{bmatrix} \quad 5. \text{略}$$

$$6. \text{負定} \quad 7. (1) (3)$$

$$8. (1) y_1^2 + 6y_2^2 \quad (2) -3y_2^2 + 3y_3^2$$

$$9. (1) \frac{x''^2}{6} + \frac{y''^2}{3} = 1 \text{ 爲橢圓} \quad (2) \frac{x''^2}{12} - \frac{y''^2}{8} = 1 \text{ 爲雙曲線}$$

第八章

習題 8-1

$$1. \text{略} \quad 2. \text{略} \quad 3. \text{略} \quad 4. \text{略}$$

$$5. \mathbf{x}(t) = c_1 e^{2t} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} \frac{1}{3} \\ t \end{bmatrix} \quad 6. \mathbf{x}(t) = c_1 e^{3t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$5. \mathbf{x}(t) = c_1 e^{2t} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} \frac{1}{3} \\ t \end{bmatrix} \quad 6. \mathbf{x}(t) = c_1 \begin{bmatrix} 2e^{3t} \\ e^{3t} \end{bmatrix} + c_2 \begin{bmatrix} -2e^{-t} \\ e^{-t} \end{bmatrix}$$

$$7. \mathbf{x}(t) = c_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -\cos t \\ \frac{1}{2}(\cos t - \sin t) \\ \cos t + \sin t \end{bmatrix} + c_3 \begin{bmatrix} -\sin t \\ \frac{1}{2}(\cos t + \sin t) \\ \sin t - \cos t \end{bmatrix}$$

習題 8-2

$$1. \mathbf{x}(t) = \begin{bmatrix} \frac{1}{6}e^{-t} + \frac{1}{2}e^t + \frac{1}{3}e^{2t} \\ -\frac{1}{6}e^{-t} + \frac{1}{2}e^t + \frac{2}{3}e^{2t} \\ \frac{1}{6}e^{-t} + \frac{1}{2}e^t + \frac{4}{3}e^{2t} \end{bmatrix} \quad 2. \mathbf{x}(t) = \begin{bmatrix} -\frac{1}{3}e^t + \frac{4}{3}e^{4t} \\ -\frac{1}{3}e^t - \frac{2}{3}e^{4t} \end{bmatrix}$$

$$3. \mathbf{x}(t) = \begin{bmatrix} e^{2t} - 2e^{3t} \\ e^t - 2e^{2t} + 2e^{3t} \\ -2e^{2t} + 2e^{3t} \end{bmatrix} \quad 4. \mathbf{x}(t) = c_1 e^{-3t} \begin{bmatrix} 3 \\ 1 \end{bmatrix} + c_2 e^{-3t} \left(\begin{bmatrix} -\frac{1}{2} \\ 0 \end{bmatrix} + t \begin{bmatrix} 3 \\ 1 \end{bmatrix} \right)$$

$$5. \mathbf{x}(t) = \begin{bmatrix} \frac{2}{3}e^{2t} + \frac{1}{3}e^{8t} \\ \frac{2}{3}e^{2t} + \frac{1}{3}e^{8t} \\ -\frac{4}{3}e^{2t} + \frac{1}{3}e^{8t} \end{bmatrix} \quad 6. \mathbf{x}(t) = \begin{bmatrix} -\frac{9}{8}e^{-t} - \frac{1}{56}e^{7t} \\ \frac{3}{4}e^{-t} - \frac{1}{28}e^{7t} \end{bmatrix}$$

$$7. \mathbf{x}(t) = \begin{bmatrix} e^{2t} + te^{2t} \\ (1-t)e^{2t} \\ e^{2t} \end{bmatrix} \quad 8. \mathbf{x}(t) = \begin{bmatrix} \frac{1}{6}e^{-2t} + \frac{5}{6}e^{4t} \\ -\frac{1}{2}e^{-2t} + \frac{5}{2}e^{4t} \end{bmatrix}$$

$$9. \mathbf{x}(t) = \begin{bmatrix} e^t \\ e^{-2t} \\ -e^{-2t} \end{bmatrix}$$

習題 8-3

$$1. \mathbf{x} = \begin{bmatrix} -2 \sin 2t \\ \cos 2t - 2 \sin 2t \end{bmatrix}$$

$$2. \mathbf{x} = c_1 \begin{bmatrix} 10 \cos 3t \\ -\cos 3t - 3 \sin 3t \end{bmatrix} + c_2 \begin{bmatrix} 10 \sin 3t \\ 3 \cos 3t - \sin 3t \end{bmatrix}$$

$$3. \mathbf{x} = c_1 \begin{bmatrix} 0 \\ 0 \\ e^{-t} \end{bmatrix} + c_2 \begin{bmatrix} \cos t - \sin t \\ -\cos t \\ \cos t \end{bmatrix} + c_3 \begin{bmatrix} \cos t + \sin t \\ -\sin t \\ \sin t \end{bmatrix}$$

$$4. \mathbf{x} = c_1 e^t \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + c_2 e^t \begin{bmatrix} -2 \sin 2t \\ \cos 2t \\ 3 \cos 2t \end{bmatrix} + c_3 e^t \begin{bmatrix} 2 \cos 2t \\ \sin 2t \\ 3 \sin 2t \end{bmatrix}$$

$$5. \mathbf{x} = 2e^{-t} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \cos 2t \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} + \sin 2t \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

習題 8-4

$$1. \mathbf{x}(t) = c_1 e^{-t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + e^t \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$2. \mathbf{x}(t) = c_1 e^{-t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} -5 \\ 1 \end{bmatrix}$$

$$3. \mathbf{x}(t) = c_1 e^t \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -\cos t \\ \frac{1}{2}(\cos t - \sin t) \\ \cos t + \sin t \end{bmatrix} + c_3 \begin{bmatrix} -\sin t \\ \frac{1}{2}(\cos t + \sin t) \\ -\cos t + \sin t \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \\ -4 \end{bmatrix}$$

$$4. \mathbf{x}(t) = c_1 e^{-t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + e^{-t} \begin{bmatrix} -\frac{1}{2} - t \\ \frac{1}{2} t \end{bmatrix}$$

$$5. \mathbf{x}(t) = c_1 e^t \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -\cos t \\ \frac{1}{2}(\cos t - \sin t) \\ \cos t + \sin t \end{bmatrix} + c_3 \begin{bmatrix} -\sin t \\ \frac{1}{2}(\cos t + \sin t) \\ -\cos t + \sin t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -e^{-t} \end{bmatrix}$$

$$6. \mathbf{x}(t) = \frac{1}{28} e^{(3+\sqrt{7})t} \begin{bmatrix} 7-\sqrt{7} \\ 2\sqrt{7}-7 \end{bmatrix} - \frac{1}{28} e^{(3-\sqrt{7})t} \begin{bmatrix} 7-\sqrt{7} \\ 2\sqrt{7}-7 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} -1 \\ t+1 \end{bmatrix}$$

$$7. \mathbf{x}(t) = c_1 e^t \begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 e^{5t} \begin{bmatrix} 1 \\ 3 \end{bmatrix} + e^t \begin{bmatrix} \frac{1}{2} \\ \frac{3}{2} \end{bmatrix} + t e^t \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$8. \mathbf{x}(t) = \begin{bmatrix} -\frac{8}{3}e^{4t} + \frac{2}{3}e^t + 3e^{5t} \\ -\frac{4}{3}e^{4t} - \frac{2}{3}e^t + e^{5t} \end{bmatrix}$$

$$9. \mathbf{x}(t) = c_1 e^t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 e^{4t} \begin{bmatrix} 2 \\ -3 \\ -3 \end{bmatrix} + c_3 e^{-2t} \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} + e^{-t} \begin{bmatrix} -\frac{1}{5} \\ -\frac{6}{5} \\ \frac{4}{5} \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$$10. \mathbf{x}(t) = \begin{bmatrix} \frac{9}{4}e^{2t} - \frac{t}{2} - \frac{1}{4} \\ -\frac{9}{10}e^{-3t} + \frac{3}{10}\sin t - \frac{1}{10}\cos t \end{bmatrix}$$

$$11. \mathbf{x}(t) = e^t \begin{bmatrix} 1+t \\ 1 \\ 0 \end{bmatrix} \quad 12. \mathbf{x}(t) = \begin{bmatrix} 1 - e^t + 2te^t \\ e^t + 2te^t \end{bmatrix} \quad 13. \mathbf{x}(t) = \begin{bmatrix} -\cos t \\ \sin t \end{bmatrix}$$

習題 8-5

1. $x(t) = 3 + 2 \cos t - 2 \sin t$
2. $x(t) = \frac{11}{6} e^{2(t-1)} + \frac{1}{6} e^{-4(t-1)}$
3. $x(t) = \frac{1}{4} (e^t + e^{-t} + 2 \cos t)$
4. $x(t) = 1 - 2e^{-t} + e^{-2t}$

第九章

習題 9-1

1. (1) $\cos ti - \sin tj$ (2) 略 (3) $t = n\pi, n=0, \pm 1, \pm 2, \dots$
(4) 是, $\sqrt{2}$ (5) 是, 1 (6) $-\sin ti - \cos tj$
2. (1) $\cos ti - e^{-t}j + k$ (2) $2i + 3t^2j$
(3) $(\cos t + 2t)i + (\cos t + 2t)j + 3(\cos t + 2t)k$
(4) $2ti - (5t^4 + e^t)j + (3t^2 - e^t)k$ (5) 0
3. (1) $\frac{8t}{\sqrt{8t^2+1}}$ (2) $6t - 10t \sin t - 5t^2 \cos t$ (3) $1 - 12t^3$ 4. 略 5. 略
6. $(1 + \sin t)i - \cos tj$ 7. 略 8. $(t^4 + 2)i - (t^2 + 4)j$

$$9. 2(t-1)\mathbf{i} + \frac{1}{2} \ln\left(\frac{t^2+1}{2}\right)\mathbf{j} + \frac{1}{2}(t^2-1)\mathbf{k}$$

習題 9-2

$$1. \kappa = \frac{1}{2}, \tau = -\frac{1}{2} \quad 2. 2\pi\sqrt{a^2+c^2}$$

$$3. \mathbf{T}\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{10}}(-\mathbf{j}+3\mathbf{k}), \mathbf{N}\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{10}}(3\mathbf{i}+\mathbf{k}), \mathbf{B}\left(\frac{\pi}{4}\right) = \mathbf{i},$$

$$\kappa\left(\frac{\pi}{4}\right) = \frac{3}{5\sqrt{5}}, \tau\left(\frac{\pi}{4}\right) = 0$$

$$4. \mathbf{T} = \frac{-a \sin t \mathbf{i} + a \cos t \mathbf{j} + c \mathbf{k}}{\sqrt{a^2+c^2}}, \mathbf{N} = -\cos t \mathbf{i} - \sin t \mathbf{j},$$

$$\mathbf{B} = \frac{c \sin t \mathbf{i} - c \cos t \mathbf{j} + a \mathbf{k}}{\sqrt{a^2+c^2}}, \kappa = \frac{a}{a^2+c^2}, \tau = \frac{c}{a^2+c^2}$$

5. 略 6. 略

$$7. \frac{ds}{dt} = \sqrt{3}e^t, a_t = \sqrt{3}e^t, a_n = \sqrt{2}e^t, \mathbf{T} = \frac{\sqrt{3}}{3}[(\cos t - \sin t)\mathbf{i} + (\sin t + \cos t)\mathbf{j} + \mathbf{k}],$$

$$\kappa = \frac{\sqrt{2}}{3}e^{-t}$$

$$8. \frac{ds}{dt} = \sqrt{5}t, a_t = \sqrt{5}, a_n = t, \kappa = \frac{1}{5t}$$

習題 9-3

$$1. (1) yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k} \quad (2) -e^{-x}\cos(yz)\mathbf{i} - ze^{-x}\sin(yz)\mathbf{j} - ye^{-x}\sin(yz)\mathbf{k}$$

$$(3) \frac{y^2-x^2}{(x^2+y^2)^2}\mathbf{i} - \frac{2xy}{(x^2+y^2)^2}\mathbf{j}$$

$$2. \text{略} \quad 3. \nabla \ln \|\mathbf{R}\| = \frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{x^2+y^2+z^2}, \nabla\left(\frac{1}{\|\mathbf{R}\|}\right) = \frac{-x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{(x^2+y^2+z^2)^{3/2}}$$

$$4. \frac{37}{3} \quad 5. \text{略} \quad 6. \frac{22}{\sqrt{21}}$$

$$7. 3x - 6y + 2z + 18 = 0, \frac{x+2}{-1} = \frac{y-1}{2} = \frac{z+3}{-2/3}$$

$$8. \text{略} \quad 9. \cos^{-1}\left(\frac{8}{3\sqrt{21}}\right) \approx 54.5^\circ$$

習題 9-4

1. 略 2. 略 3. $2(x+1)\mathbf{j}$ 4. 略 5. 略 6. 0

習題 9-5

1. (1) $\frac{4}{3}$ (2) $\frac{4}{3}$ (3) 0 2. $\sqrt{2}\pi$ 3. $-\frac{8}{3}$
4. 0 5. (1) $\frac{5}{3}$ (2) $\frac{8}{3}$ (3) 2 6. $1-e^3$ 7. $\frac{3}{5}$
8. $-\frac{7}{6}$ 9. (1) 5 (2) $\frac{23}{3}$ (3) $\frac{13}{3}$ 10. 18π

習題 9-6

1. 略 2. 略 3. 略 4. 略 5. 略
6. (1) -4 (2) -4 (3) -4 (4) 0 7. $1-\frac{1}{e}$
8. (1) 略 (2) $\phi(x, y, z) = y^2 \sin x + xz^3 - 4y + 2z + c$ (3) $4\pi + 15$ 9. 略

習題 9-7

1. 0 2. 0 3. 0 4. 9π 5. $\frac{1}{30}$ 6. π
7. -12π 8. 0 9. (1) πab (2) πab

習題 9-8

1. 90 2. 11 3. $2\pi r^3$ 4. 24 5. $\frac{3}{2}$ 6. 1

習題 9-9

1. 3V 2. 84π 3. 略 4. 略 5. 略

第十章

習題 10-1

1. $\frac{2}{\pi} + \sum_{n=1}^{\infty} \frac{4 \cos n\pi}{\pi(1-4n^2)} \cos nx$

2. $\frac{\sinh \pi}{\pi} \left\{ 1 + 2 \sum_{n=1}^{\infty} \left[\frac{(-1)^n}{n^2+1} \cos nx + \frac{n(-1)^{n+1}}{n^2+1} \sin nx \right] \right\}$
3. $\frac{\pi}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos n\pi - 1}{n^2} \cos nx$
4. $\frac{\pi^2}{6} + \sum_{n=1}^{\infty} \left\{ \frac{2(-1)^n}{n^2} \cos nx + \frac{2[(-1)^n - 1] - n^2 \pi^2 (-1)^n}{n^3 \pi} \sin nx \right\}$
5. $\frac{1}{4} + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left[\sin \frac{n\pi}{2} \cos nx + \left(1 - \cos \frac{n\pi}{2} \right) \sin nx \right]$
6. $\frac{12}{\pi^3} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin n\pi x}{n^3}$
7. $-\frac{1}{\pi} + \frac{1}{2} \cos x + \frac{1}{2} \sin x + \frac{2}{3\pi} \cos 2x + \frac{4}{3\pi} \sin 2x + \frac{2}{15\pi} \cos 4x + \frac{8}{15\pi} \sin 4x + \dots$
8. $\frac{20}{\pi} \left(\sin x + \frac{1}{3} \sin 3x + \frac{6}{5} \sin 5x + \dots \right)$

習題 10-2

1. (1) 奇函數 (2) 偶函數 (3) 奇函數 (4) 偶函數
(5) 兩者皆非 (6) 偶函數 (7) 偶函數

2. $\frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left(1 - \cos \frac{n\pi}{2} \right) \sin n\pi x$
3. $2\pi \sum_{n=1}^{\infty} \frac{n(1 - e \cos n\pi)}{n^2 \pi^2 + 1} \sin n\pi x$
4. $\frac{8}{\pi} \sum_{n=1}^{\infty} (-1)^n \frac{n}{1 - 4n^2} \sin n\pi x$
5. $\frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n\pi}{2} \cos n\pi x$
6. $\frac{4}{3} + \frac{16}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos \frac{n\pi x}{2}$
7. $\frac{\sin 2}{2} + 4 \sin 2 \left[\sum_{n=1}^{\infty} \frac{(-1)^n}{4 - n^2 \pi^2} \cos \frac{n\pi x}{2} \right]$

習題 10-3

1. $y_n(t) = \frac{F_0}{9} + A_n \cos \frac{2n\pi t}{3} + B_n \sin \frac{2n\pi t}{3}, n=1, 2, 3, \dots$

$$A_n = \frac{F_0}{n\pi D_n} \left[\sin \frac{2n\pi}{3} \left(3 - \frac{32n^2 \pi^2}{9} \right) + \frac{0.2n\pi}{3} \left(1 - \cos \frac{2n\pi}{3} \right) \right]$$

$$B_n = \frac{F_0}{n\pi D_n} \left[\left(1 - \cos \frac{2n\pi}{3} \right) \left(3 - \frac{32n^2 \pi^2}{9} \right) + \frac{0.2n\pi}{3} \sin \frac{2n\pi}{3} \right]$$

$$D_n = \left(3 - \frac{32n^2\pi^2}{9}\right)^2 - \frac{0.04n^2\pi^2}{9}$$

$$2. y_n(t) = \frac{0.2(-1)^n}{D_n} \cos 2n\pi t + \frac{40(-1)^n(10n^2\pi^2 - 1)}{n\pi D_n} \sin 2n\pi t, \quad n=1, 2, 3, \dots$$

$$D_n = (400n^2\pi^2 - 40)^2 + 0.04n^2\pi^2$$

$$3. I_n(t) = A_n \cos 200n\pi t + B_n \sin 200n\pi t, \quad n=1, 2, 3, \dots$$

$$A_n = \frac{-0.0002D_n}{D_n^2 + 144n^2\pi^2}, \quad B_n = \frac{0.0014n\pi}{D_n^2 + 144n^2\pi^2}, \quad D_n = 2.5 - 4n^2\pi^2$$

$$4. I_n(t) = A_n \cos 100n\pi t + B_n \sin 100n\pi t, \quad n=1, 2, 3, \dots$$

$$A_n = \frac{100n^2\pi}{(1 - 4n^2)(D_n^2 + 25n^2\pi^2)}, \quad B_n = \frac{20nD_n}{(4n^2 - 1)(D_n^2 + 25n^2\pi^2)},$$

$$D_n = 50 - 2n^2\pi^2$$

第十一章

習題 11-1

$$1. \int_0^\infty \frac{20}{\pi\lambda} \sin(10\lambda) \cos(\lambda t) d\lambda$$

$$2. \int_0^\infty \left[\frac{2 \sin(\lambda\pi)}{\pi\lambda^2} - \frac{2}{\lambda} \cos(\lambda\pi) \right] \sin(\lambda t) d\lambda$$

$$3. \frac{2}{\pi} \int_0^\infty \frac{1 - \cos \lambda}{\lambda} \sin(\lambda t) d\lambda \quad (2) \quad \frac{2}{\pi} \int_0^\infty \frac{\sin \lambda}{\lambda} \cos(\lambda t) d\lambda$$

$$4. f(t) = \frac{2}{\pi} \int_0^\infty \frac{1 - \cos \lambda}{\lambda^2} \cos \lambda t d\lambda, \quad \frac{\pi}{2}$$

$$5. \frac{4}{\pi} \int_0^\infty \left(\frac{\sin \lambda}{\lambda} + \frac{\cos \lambda - 1}{\lambda^2} \right) \cos(\lambda t) d\lambda$$

$$6. (1) \frac{2}{\pi\lambda^2} \int_0^\infty (\sin 2\lambda - 3\lambda \cos 2\lambda + \lambda \cos \lambda) \sin(\lambda x) d\lambda$$

$$(2) \frac{2}{\pi\lambda^2} \int_0^\infty (\cos 2\lambda - 1 + 3\lambda \sin 2\lambda - \lambda \sin \lambda) \cos(\lambda x) d\lambda \quad 7. \text{略}$$

習題 11-2

$$1. 3 + \frac{3i}{\pi} \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{1}{n} e^{\frac{n\pi i x}{4}}, \text{ 收斂到 } f(x), \text{ 其中 } 0 < x < 8, 8 < x < 16, \dots \text{ 及 } -8 < x < 0,$$

$$2. 3 + \frac{3i}{\pi} \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{1}{n} e^{\frac{2n\pi ix}{3}}, \text{ 收斂到 } \begin{cases} 3, & x=0 \text{ 或 } x=3 \\ 2x, & 0 < x < 3 \end{cases}$$

$$3. \frac{4}{3} + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \left(\frac{2}{n^2 \pi^2} - \frac{2i}{n\pi} \right) e^{n\pi ix}, \text{ 收斂到 } \begin{cases} 2, & x=0 \text{ 或 } x=2 \\ x^2, & 0 < x < 2 \end{cases}$$

$$4. - \sum_{n=-\infty}^{\infty} \frac{2i}{n\pi} e^{\frac{inx}{2}}, \quad n=1, 3, 5, 7, \dots$$

$$5. f(x) = \sin(\pi x)$$

$$6. f(x) = -\frac{2i}{\pi} \int_{-\infty}^{\infty} \frac{\lambda}{(1+\lambda^2)^2} e^{i\lambda x} d\lambda, \quad \forall x \in \mathbb{R}$$

$$7. (1) \frac{4}{\pi} \int_0^{\infty} \frac{\sin \lambda}{\lambda} \cos(\lambda t) d\lambda \quad (2) \frac{\pi}{4}$$

習題 11-3

$$1. F(\omega) = i \left[\frac{\sin(2(\omega+1))}{\omega+1} - \frac{\sin(2(\omega-1))}{\omega-1} \right]$$

$$2. F(\omega) = \frac{2i(\cos \omega - 1)}{\omega}$$

$$3. F(\omega) = \frac{3e^{-3(3+i\omega)}}{3+i\omega}$$

$$4. F(\omega) = \frac{4e^{-2(1/4+i\omega)}}{1+4i\omega}$$

$$5. F(\omega) = \frac{i\pi}{9} (e^{-3|\omega+1|} - e^{-3|\omega-1|})$$

$$6. F(\omega) = \frac{12}{16+(\omega+2)^2} + \frac{12}{16+(\omega-2)^2}$$

$$7. F(\omega) = 2\sqrt{3}\pi e^{-\omega(\omega/12+6i)}$$

$$8. F(\omega) = \frac{\pi}{2} e^{3i\omega-2|\omega|}$$

$$9. F(\omega) = \pi i [u(-\omega)e^{5\omega} - u(\omega)e^{-5\omega}]$$

$$10. F(\omega) = -\frac{\sqrt{\pi}i\omega}{2} e^{-\omega^2/4}$$

$$11. F(\omega) = \frac{5}{(4+i\omega)^2}$$

$$12. F(\omega) = \frac{-i\omega\sqrt{\pi}}{18} e^{-\omega^2/36}$$

$$13. F_s(\omega) = \frac{\omega}{k^2+\omega^2}, \quad F_c(\omega) = \frac{k}{k^2+\omega^2}$$

$$14. F_s(\omega) = \frac{\omega}{1+\omega^2}, \quad F_c(\omega) = \frac{1}{1+\omega^2}$$

$$15. F_s(\omega) = \frac{2\omega}{(1+\omega^2)^2}, \quad F_c(\omega) = \frac{1-\omega^2}{(1+\omega^2)^2}$$

$$16. F_s(\omega) = \frac{1}{\omega} (1 - 2 \cos \omega + \cos 2\omega), \quad F_c(\omega) = \frac{1}{\omega} (2 \sin \omega - \sin 2\omega)$$

$$17. f(t) = e^{6it} u(t-4) e^{-4(t-4)} \quad 18. f(t) = u(t) (2e^{-3t} - e^{-2t})$$

$$19. f(t) = -6e^{-\pi i t} [u(t+5) - u(t-5)]$$

$$20. \frac{1}{4} u(t+5) [1 - e^{-2(t+5)}] - \frac{1}{4} u(t-5) [1 - e^{-2(t-5)}]$$

$$21. y(t) = \frac{1}{4} [u(t-1)e^{-(t-1)} - u(t-1)e^{-5(t-1)}] \quad 22. \frac{2(1 - \cos t)}{\pi t^2}$$

$$23. f(t) = \frac{2}{\pi(1+t^2)}$$

第十二章

習題 12-1

1. 略 2. 略

3. (1) 橢圓 (2) 圓 (3) 雙曲線 (4) 雙曲線 (5) 圓

4. (1) $|z+3-4i|=2$ (2) $|z-2i|+|z+2i|=10$

5. (1) 略 (2) 略 6. (1) $-16(1+\sqrt{3}i)$ (2) $-\frac{\sqrt{3}}{2} - \frac{1}{2}i$ 7. 略

8. 略 9. (1) 略 (2) 略 10. (1) 略 (2) 略

$$11. (1) z_1 = \sqrt[6]{2} \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right), \quad z_2 = \sqrt[6]{2} \left(\cos \frac{11\pi}{12} + i \sin \frac{11\pi}{12} \right),$$

$$z_3 = \sqrt[6]{2} \left(\cos \frac{19\pi}{12} + i \sin \frac{19\pi}{12} \right)$$

$$(2) z_1 = \sqrt{2} \left(\cos \frac{7\pi}{24} + i \sin \frac{7\pi}{24} \right), \quad z_2 = \sqrt{2} \left(\cos \frac{19\pi}{24} + i \sin \frac{19\pi}{24} \right),$$

$$z_3 = \sqrt{2} \left(\cos \frac{31\pi}{24} + i \sin \frac{31\pi}{24} \right), \quad z_4 = \sqrt{2} \left(\cos \frac{43\pi}{24} + i \sin \frac{43\pi}{24} \right),$$

$$12. -\frac{3}{2} \pm \frac{\sqrt{7}}{2}i, \quad \frac{3}{2} \pm \frac{\sqrt{7}}{2}i \quad 13. 2-3i, \quad 1+i$$

習題 12-2

1. (1) $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ (2) $-12 + 6i$ (3) $\frac{1}{3}$ (4) $-\frac{4}{3} - 4i$

(5) $\frac{1}{6} - \frac{\sqrt{3}}{6}i$ (6) $-\frac{1}{4}$

2. (1) $5z^4 + 6z^2 + 1$ (2) $20z^3 - 2z^2 - 8$ (3) $2z - \frac{2}{z^3}$ (4) $\frac{-z^4 - 10z^2 - 10z - 8}{(z^3 + 2z - 5)^2}$

3. 略 4. 略 5. 略

6. (1) $\frac{5}{3}$ (2) $-\frac{1}{4}$ (3) $\frac{4}{25}(4 + 3i)$ 7. 略

8. (1) 略 (2) $v = e^{-x}(y \sin y + x \cos y) + c$

習題 12-3

1. (1) $\frac{1}{2}$ (2) $\frac{1}{2}$ (3) $\frac{1}{6}(1 - \sqrt{3}i)$ (4) $\frac{1}{6}$

2. (1) $\frac{1}{2} \ln 2 - \frac{\pi}{4}i$ (2) $\frac{1}{4}[2 \ln 2 + \pi + (\pi - 2 \ln 2)i]$

3. (1) $e^{-3\pi/4 + (i/2) \ln 2}$ (2) $e^{-\pi/4 + (i/2) \ln 2}$ (3) $e^{\pi/4 + (i/2) \ln 2}$

4. (1) $z = -i \ln(2 \pm \sqrt{3}) + 2k\pi, k=0, \pm 1, \pm 2, \dots$ (2) $\frac{\pi i}{8}$

習題 12-4

1. $\int_C (z - z_0)^n dz = \begin{cases} 2\pi i, & n = -1 \\ 0, & n \neq -1 \end{cases}$ 2. $10 - 8i$ 3. 8 4. 0

5. (1) $\frac{1}{3} \ln(z^3 + 3z + 2) + c$ (2) $-\frac{1}{2} \cos z^2 + c$

(3) $\frac{1}{2} z \sin 2z + \frac{1}{4} \cos 2z + c$ (4) $\frac{1}{10} \sin^5 2z + c$

(5) $\frac{z^2}{2} \ln z - \frac{z^2}{4} + c$ (6) $-e^{-z}(z^2 + 2z + 2) + c$

6. (1) i (2) $2i$ 7. (1) $-2\pi i$ (2) 0

8. 0 9. $-\pi i$ 10. (1) 0 (2) $-\frac{1}{2}$

11. 略 12. (1) $4\pi i$ (2) $\frac{8}{3}\pi i e^{-2}$ 13. 略 14. 略 15. $\frac{1}{4}$

習題 12-5

1. (1) $\frac{1}{z^2} - \frac{4}{z^3} + \frac{13}{z^4} - \frac{40}{z^5} + \dots$
 (2) $\dots - \frac{1}{2z^4} + \frac{1}{2z^3} - \frac{1}{2z^2} + \frac{1}{2z} - \frac{1}{6} + \frac{z}{18} - \frac{z^2}{54} + \frac{z^3}{162} - \dots$
 (3) $\frac{1}{3} - \frac{4}{9}z + \frac{13}{27}z^2 - \frac{40}{81}z^3 + \dots$
 (4) $\frac{1}{2(z+1)} - \frac{1}{4} + \frac{1}{8}(z+1) - \frac{1}{16}(z+1)^2 + \dots$
2. (1) $\dots + \frac{1}{z^2} + \frac{1}{z} + 1 + \frac{z}{2} + \frac{z^2}{4} + \frac{z^3}{8} + \dots$
 (2) $-\frac{1}{2}z - \frac{3}{4}z^2 - \frac{7}{8}z^3 - \frac{15}{16}z^4 - \dots$
 (3) $-\frac{1}{z-1} - \frac{2}{(z-1)^2} - \frac{2}{(z-1)^3} - \dots$
 (4) $-\frac{1}{2} - \frac{3}{z^2} - \frac{7}{z^3} - \frac{15}{z^4} - \dots$
 (5) $1 - \frac{2}{z-2} - (z-2) + (z-2)^2 - (z-2)^3 + (z-2)^4 - \dots$
3. (1) $-\frac{1}{2z^2} - \frac{1}{4z} + \frac{1}{3} \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} - \frac{1}{24} \sum_{n=0}^{\infty} (-1)^n \frac{z^n}{2^n}$
 (2) $-\frac{1}{2z^2} - \frac{1}{4z} - \frac{1}{3} \sum_{n=0}^{\infty} z^n - \frac{1}{24} \sum_{n=0}^{\infty} (-1)^n \frac{z^n}{2^n}$
 (3) $-\frac{1}{2z^2} - \frac{1}{4z} + \frac{1}{3} \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} - \frac{1}{12} \sum_{n=0}^{\infty} (-1)^n \frac{2^n}{z^{n+1}}$
4. (1) $\frac{2+i}{10} \sum_{n=0}^{\infty} (iz)^n + \frac{2-i}{10} \sum_{n=0}^{\infty} (-1)^n (iz)^n + \frac{1}{10} \sum_{n=0}^{\infty} (-1)^n \frac{z^n}{2^n}$
 (2) $\frac{-1+2i}{10} \sum_{n=0}^{\infty} \frac{(-1)^n i^n}{z^{n+1}} - \frac{1+2i}{10} \sum_{n=0}^{\infty} \frac{i^n}{z^{n+1}} + \frac{1}{5} \sum_{n=0}^{\infty} (-1)^n \frac{2^n}{z^{n+1}}$
 (3) $\frac{-1+2i}{10} \sum_{n=0}^{\infty} \frac{(-1)^n i^n}{z^{n+1}} - \frac{1+2i}{10} \sum_{n=0}^{\infty} \frac{i^n}{z^{n+1}} + \frac{1}{10} \sum_{n=0}^{\infty} (-1)^n \frac{z^n}{2^n}$
5. (1) $1 + \frac{3}{2} \sum_{n=0}^{\infty} (-1)^n \frac{z^n}{2^n} - \frac{8}{3} \sum_{n=0}^{\infty} (-1)^n \frac{z^n}{2^n}$

$$(2) \ 1 + 3 \sum_{n=0}^{\infty} (-1)^n \frac{2^n}{z^{n+1}} - 8 \sum_{n=0}^{\infty} (-1)^n \frac{3^n}{z^{n+1}}$$

$$(3) \ 1 + 3 \sum_{n=0}^{\infty} (-1)^n \frac{2^n}{z^{n+1}} - \frac{8}{3} \sum_{n=0}^{\infty} (-1)^n \frac{z^n}{3^n}$$

習題 12-6

$$1. \ 2\pi i \left(\frac{t-1}{2} + \frac{1}{2} e^{-t} \cos t \right) \quad 2. \ -\frac{2\pi i}{3}$$

$$3. \ (1) \ \frac{\pi}{5} \quad (2) \ 0 \quad (3) \ -\frac{\pi}{5} \quad (4) \ \frac{2\pi i}{5} \quad (5) \ -\frac{2\pi i}{5}$$

$$4. \ (1) \ 2\pi i \quad (2) \ 6\pi i \quad 5. \ \frac{\pi i}{3}$$

習題 12-7

$$1. \ \frac{5\pi}{32} \quad 2. \ \frac{\pi}{12} \quad 3. \ \frac{2\pi}{\sqrt{a^2-b^2}} \quad 4. \ \frac{2\pi}{\sqrt{1-a^2}} \quad 5. \ \frac{\pi}{3} \quad 6. \ \pi$$

$$7. \ \frac{7\pi}{50} \quad 8. \ \frac{\pi}{3} \quad 9. \ -\pi e^{-2\pi} \quad 10. \ \frac{\pi(e-1)}{6e^2} \quad 11. \ \text{略} \quad 12. \ \text{略}$$

習題 12-8

$$1. \ \text{略} \quad 2. \ (1) \ v = \frac{1}{2}(1-u^2) \quad (2) \ w\text{-平面的上半平面} \quad 3. \ w = \frac{i-z}{i+z}$$

$$4. \ w = i - \left(\frac{z+1}{z-1} \right) \quad 5. \ \text{略}$$

十三章

$$1. \ (1) \ \text{線性} \quad (2) \ \text{非線性} \quad (3) \ \text{非線性} \quad (4) \ \text{非線性}$$

$$(5) \ \text{線性} \quad (6) \ \text{線性} \quad 2. \ \text{略}$$

$$3. \ u(x, y) = 3xy + \frac{y^2}{2} + f(x)$$

$$4. \ u(x, y) = -y \cos x + f(y)$$

$$5. \ u(x, y) = \ln x + xe^y + f(y)$$

$$6. \ u(x, y) = -e^{x-y} + f(x) + g(y)$$

$$7. \ u(x, y) = ye^x + xf(y) + g(y)$$

習題 13-1

$$8. u(x, y) = 2 \left(x - \frac{1}{y} \right) + e^{-xy} f(y)$$

$$9. u(x, y) = -\frac{\cos x}{y} + y^2 f(x)$$

$$10. u(x, y) = f(y) e^{2xy} + g(y) e^{-2xy} - \frac{3x}{4y^2}$$

$$11. u(x, y) = f(y) e^{xy} + g(y) e^{-3xy}$$

$$12. u(x, y) = f(x) e^{xy} + g(x) y e^{xy}$$

$$13. u(x, y) = x e^y + \frac{f(y)}{x} + g(x)$$

$$14. u(x, y) = 1 - \cos x + y^2 + \frac{x^2 y^2}{4}$$

$$15. u(x, y) = x - y + e^y - 1 + \frac{x^3 \sin y}{3}$$

習題 13-2

$$1. F\left(\frac{z}{x^3}, \frac{y}{x}\right) = 0 \quad 2. F(y^2 + z^2, x^3 + y^3) = 0 \quad 3. F(x^2 + 2yz, x + y + z) = 0$$

$$4. F\left(\frac{y}{z}, \frac{x^2 + y^2 + z^2}{z}\right) = 0 \quad 5. z = e^{-cx/a} \phi(ay - bx) \text{ 或 } z = e^{-cy/b} \psi(ay - bx)$$

$$6. (az - cx)^2 + (bz - cy)^2 = r^2(z - c)^2 \quad 7. 2xy + z(x + y) = 3xyz$$

$$8. F(u, x + y + z, x^2 + y^2 + z^2) = 0$$

習題 13-3

1. (1) 雙曲線型 (2) 橢圓型 (3) 拋物線型 (4) 橢圓型 (5) 橢圓型

$$2. (1) x^2 + y^2 > 1 \quad (2) x^2 + y^2 = 1 \quad (3) x^2 + y^2 < 1$$

$$3. (1) x^2 - 4y > 0 \quad (2) x^2 - 4y = 0 \quad (3) x^2 - 4y < 0$$

$$4. u(x, y) = e^{3x} F(y - 2x) \quad 5. u(x, y) = F(y - x) + G(y - 2x)$$

$$6. u(x, y) = F(3x + y) + G(y - x)$$

$$7. u(x, y) = F(x + 2y) + xG(x + 2y) \text{ 或 } u(x, y) = F(x + 2y) + yG(x + 2y)$$

$$8. u(x, y) = F(x + y) + xG(x + y) \quad 9. u(x, y) = F(x - y) + G(x + y) - y^4$$

$$10. u(x, y) = F(2x + y) + G(2x - y) + \frac{1}{4} x e^{2x + y}$$

習題 13-4

$$1. u(x, t) = \frac{3}{4} \sin x \cos t - \frac{1}{4} \sin 3x \cos 3t$$

$$2. u(x, t) = \sum_{n=1}^{\infty} B_n \sin nct \sin nx, \quad B_n = \frac{2[1 + (-1)^{n+1} e^{\pi}]}{c\pi(n^2 + 1)}$$

$$3. u(x, t) = \frac{5}{2\pi} \sin \pi x \sin 2\pi t$$

$$4. u(x, t) = \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{e^{-(2n-1)^2 t}}{(2n-1)^3} \sin (2n-1)x$$

$$5. u(x, t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{e^{-k(2n-1)^2 t}}{(2n-1)^2} \sin (2n-1)x$$

$$6. u(x, y) = \sum_{n=1}^{\infty} \left[a_n \frac{\sinh n(\pi-y)}{\sinh n\pi} \sin nx + b_n \frac{\sinh n(\pi-x)}{\sinh n\pi} \sin ny \right], a_2 = b_2 = 0$$

$$a_n = \frac{1}{\pi} (1 - \cos n\pi) \left[\frac{1}{n} - \frac{n}{(n-2)(n+2)} \right]$$

$$b_n = \frac{1}{\pi} (1 - \cos n\pi) \left[\frac{1}{n} + \frac{n}{(n-2)(n+2)} \right], n \neq 2$$

$$7. u(r, \theta) = \frac{3}{4} a^2 r \sin \theta - \frac{1}{4} r^3 \sin 3\theta$$

$$8. u(r, \theta) = \frac{1}{2} + \frac{1}{2} r^2 \cos 2\theta + r \sin \theta$$

$$9. u(x, y) = \frac{1}{2} (a^2 + x^2 - y^2)$$

$$10. \text{略} \quad 11. u(x, t) = \frac{4l^2}{c\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^3} \sin \frac{n\pi}{2} \sin \frac{n\pi t}{l} \sin \frac{n\pi x}{l}$$

$$12. u(x, t) = \frac{8000}{\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^3} \left(\frac{1}{n^2 \pi^2} - 12 \right) e^{-n^2 c^2 \pi^2 t} \sin \frac{n\pi x}{10}$$

$$13. u(x, y) = 800 \sum_{n=1}^{\infty} \frac{1}{n\pi \sinh (n\pi/2)} \left(\cos n\pi - \frac{4 \cos n\pi}{n^2 \pi^2} + \frac{4}{n^2 \pi^2} \right) \sin \frac{n\pi x}{20} \sinh \frac{n\pi y}{20}$$

習題 13-5

$$1. u(x, t) = 2e^{-36t} \sin 3x - 4e^{-100t} \sin 5x$$

$$2. u(x, t) = 5e^{-32\pi^2 t} \sin 4\pi x - 3e^{-128\pi^2 t} \sin 8\pi x + 2e^{-200\pi^2 t} \sin 10\pi x$$

$$3. u(x, t) = 6e^{-\pi^2 t/4} \sin \frac{\pi x}{2} + 3e^{-\pi^2 t} \sin \pi x$$

$$4. u(x, t) = \frac{3}{4\pi} \sin 2\pi x \sin 4\pi t - \frac{1}{5\pi} \sin 5\pi x \sin 10\pi t$$